Existence of globally attracting solutions of the viscous Burgers equation on the line with periodic boundary conditions and nonautonomous forcing

Jacek Cyranka
Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Poland
jcyranka@gmail.com

We prove the existence of globally attracting solutions of the viscous Burgers equation with periodic boundary conditions on the line for some particular choices of viscosity and non-autonomous forcing $u_t + u \cdot u_x - \nu u_{xx} = f(t, x)$.
The attracting solution is periodic if the forcing is periodic. The convergence towards attracting solution is exponential. The proof is computer assisted. The method is general and can be applied to other similar partial differential equations including the Navier-Stokes equations.
The technique we use is not restricted to some particular type of equation nor to the dimension one, as we are not using any maximum principles, nor unconstructive functional analysis techniques. We need some kind of 'energy' decay as a global property of our dissipative PDEs and then if the system exhibits an attracting orbit, then we should in principle be able to prove it independent of the dimensionality of of the system. At the present state our technique strongly relies on the existence of good coordinates, the Fourier modes in the considered example. We hope that the further development of the rigorous numerics for dissipative PDEs based on other function bases, e.g. for example the finite elements, should allow to treat also different domains and boundary conditions in near future.
[1] J. Cyranka, P. Zgliczyński, Existence of globally attracting solutions for one-dimensional viscous Burgers equation with nonautonomous forcing - a computer assisted proof, in revision, arXiv:1403.7170.
[2]J. Cyranka, P. Zgliczyński: The effect of fast movement in dissipative PDEs with the forcing term, preliminary preprint, arXiv:1407.1712.

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