Computer-assisted analysis of Craik's 3D dynamical system

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The following system of equations is studied:

$$\dot{x} = ayz + bz + cy, \dot{y} = dzx + ex + fy, \dot{z} = gxy + hy + kx,$$

where $x(t), y(t)$, and $z(t)$ are real-valued functions, $\dot{x}, \dot{y}$, and $\dot{z}$ are their derivatives with respect to the independent variable t, and the coefficients a to k are real constants. This system arises several contexts in mechanics and fluid mechanics. Especially, Craik has shown that the equations of the form describe a class of exact solutions of the full incompressible Navier-Stokes equations.

Most of solution orbits for the system are unbounded. We can, however, observe characteristic behaviour. A typical solution orbit draws a helical curve, which changes amplitude in a vicinity of the origin. Some solutions change only the amplitude, while some solutions change not only the amplitude but also the axis along which they go to infinity as $t \to \infty$. Craik and Okamoto have found a four-leaf structure and a periodic orbit, which play an important role in controlling the solution orbits. We prove the existence of such a periodic orbit by a method of numerical verification.