Coexistence of chaos and hyperchaos

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Chaotic attractors can have a very nonuniform internal structure. Even on the plane the well known Newhouse phenomenon guarantees that small sinks of very high periods may be embedded in large chaotic zones. For higher dimensional systems one can expect coexistence of chaotic and hyperchaotic dynamics, i.e. topological horseshoes with more than one positive Lyapunov exponents.

Consider the classical 4D Rossler system

$$\dot{x} = -y - w, \quad \dot{y} = x + ay + z, \quad \dot{z} = dz + cw, \quad \dot{w} = xw + b$$

with the parameter values $a = 0.27857$, $b = 3$, $c = -0.3$, $d = 0.05$. Let

$$\Pi = \{(x, 0, z, w) \in \mathbb{R}^3, \dot{y} = x + z < 0\}$$

be the Poincare section and let $P : \Pi \to \Pi$ be the associated Poincare map.

We prove that

1) there is an explicitly given trapping region $B \subset \Pi$ for P, i.e. $P(B) \subset B$,

2) the maximal invariant set $A = \mathrm{inv}(P, B)$ contains three invariant sets, say H_1, H_2, H_3, on which the dynamics is Σ_2 chaotic, i.e. it is semiconjugated to the Bernoulli shift on two symbols,

3) H_1 is a chaotic set with two positive Lyapunov exponents,

4) H_2 and H_3 are chaotic sets with one positive Lyapunov exponent,

5) there is a countable infinity of heteroclinic connections linking H_1 with H_2, H_2 with H_3 and H_1 with H_3,

6) there is countable infinity of periodic orbits and heteroclinic/homoclinic orbits inside each horseshoe.

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