Concurrent lines on del Pezzo surfaces of degree one

Ronald van Luijk
Universiteit Leiden, Netherlands
rmluijk@gmail.com

Let k be a field and $\overline{k}$ an algebraic closure. A del Pezzo surface over k is a surface over k that is isomorphic over $\overline{k}$ to either $\mathbb{P}^1 \times \mathbb{P}^1$ (degree 8), or $\mathbb{P}^2$ blown up at $r \leq 8$ points in general position (degree $9 - r$). Famous examples (with $r = 6$ and degree 3) are smooth cubic surfaces in $\mathbb{P}^3$, which over $\overline{k}$ contain 27 lines; at most three of these can be concurrent, that is, go through the same point. Analogously, we get 240 lines for $r = 8$ and degree 1. Based on the graph on these lines, with edges between those that intersect, we get an upper bound of 16 for the number of concurrent lines. We show that this upper bound is only attained in characteristic 2, which makes the case $r = 8$ different from all other cases. In most characteristics, including characteristic 0, the upper bound is 10.

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