Computing twists of Shioda modular surfaces of level 4 related to visibility of Sha

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One of the most mysterious objects associated to an elliptic curve E is its Tate-Shafarevich group $Sha(E)$. Its elements can be represented by classes in the Galois-cohomology group $H^1(Q, E[n])$, for various n.

If two distinct elliptic curves E and E' have isomorphic n-torsion, then a single class ξ in $H^1(Q, E[n])$ can represent a trivial element in $Sha(E)$ and a non-trivial one in $Sha(E')$. In the terminology of Mazur, the element of $Sha(E')$ is made *visible* by E. Mazur showed that for $n = 3$, all elements of Sha can be made visible. In general, the question translates into whether a rational point lies on a certain twist of the Shioda modular surface, obtained by taking the universal elliptic curve over the modular curve $X(n)$ of full level n.

The case $n = 4$ is particularly interesting. The curve $X(4)$ is rational, but the relevant surface over it is not. It is a K3 surface. Further complications in determining the correct surface arise from the fact that 4 is even. We will discuss how to compute a model of the relevant surface given ξ and give some examples of the various obstructions to rational points that can arise on these surfaces.

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