Congruences between modular forms modulo prime powers

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Consider the following problem: given a modular form f and a prime power p^n, is there a modular form g, different from f, such that f and g are congruent modulo p^n?. A way to solve this problem is to consider the Galois representation attached to f, take its modulo p^n reduction and study the obstructions appearing when trying to lift it back to a ring of characteristic zero. In this way, we obtain a local-to-global lifting result for abstract representations, which, when combined with the appropiate modularity lifting theorem, translates into an affirmative response for the proposed problem. Moreover, the method provides a way to control the local behavior of the representations (and hence the modular forms) constructed, giving applications to level lowering and level raising problems.

We can show in a concrete example how the method works, getting a case of modulo p^n level raising. We find a congruence modulo 25 between an elliptic curve of conductor 17 and a modular form of level $17 \cdot 113$. The obtainment of this example involves computing groups of Galois cohomology of the representation attached to the elliptic curve and understanding the local behavior of the elements lying inside them. For this, we compute abelian extensions of a high degree Galois extension of $\mathbb{Q}$, which turns out to be a computationally challenging problem.

Joint work with Ariel Pacetti (Universidad de Buenos Aires, Argentina).