Stability and Ramsey numbers for cycles and wheels

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We study the structure of red-blue edge-colorings of a complete graph, in such a way that certain graphs don't appear as monochromatic subgraphs. More concretely, we consider the case of an odd positive integer n, and forbidden monochromatic graphs given by a red n-cycle C_n and a blue n-wheel $W_n = C_n + K_1$.
Our main result is that if complete graph G of appropriate size has a red-blue edge-coloring of its edges in a way such that C_n is not a red subgraph of G nor W_n is a blue subgraph of G; then deleting at most two vertices we obtain a partition of the vertices of G in three sets such that the edges with both ends in the same element of the partition are colored red, and blue otherwise.
We can see this result as a generalization of results previously shown by Nikiforov and Schelp, in the case where the forbidden monochromatic subgraphs are odd cycles.
As a secondary result of our proof, we obtain two bounds for the Ramsey number of $r(C_{2k+1}, W_{2k+1})$: one is tighter for small values of k, and the other is better in the asymptotic case. The exact values for $r(C_{2k+1}, W_{2k+1})$ are presently an open problem. Our bounds are a rough approximation to the conjectured values and show that they are, at least, asymptotically true.

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