Counting k-uniform linear hypergraphs in sparse pseudorandom hypergraphs

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We give a counting lemma for the number of copies of linear k-uniform connector-free hypergraphs (connector is a generalization of triangle for hypergraphs) that are contained in some sparse hypergraphs G. Let H be a linear k-uniform connector-free hypergraph and let G be a k-uniform hypergraph with n vertices. Set $d_H = \max\{\delta(J)\colon J \subset H\}$ and $D_H = \min\{kd_H, \Delta(H)\}$. We proved that if the vertices of G do not have large degree, small families of $(k-1)$-element sets of $V(G)$ do not have large common neighbourhood and most of the pairs of sets in $\binom{V(G)}{k-1}$ have the 'right' number of common neighbours, then the number of embeddings of H in G is $(1+o(1))n^{|V(H)|}p^{|E(H)|}$, given that $p \gg n^{-1/D_H}$ and $|E(G)| = \binom{n}{k}p$. This generalizes a result by Kohayakawa, Rödl and Sissokho [Embedding graphs with bounded degree in sparse pseudo-random graphs, Israel J. Math. 139 (2004), 93-137], who proved that, for p as above, this result holds for graphs, where H is a triangle-free graph.

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