Local EPT graphs on bounded degree trees

María Pía Mazzoleni
Universidad Nacional de La Plata, CONICET, Argentina
pia@mate.unlp.edu.ar

A graph G is called an EPT graph if it is the edge intersection graph of a family of paths in a tree. An EPT representation of G is a pair $\langle \mathcal{P}, T \rangle$ where $\mathcal{P}$ is a family $(P_v)_{v \in V(G)}$ of subpaths of the host tree T satisfying that two vertices v and v' of G are adjacent if and only if P_v and $P_{v'}$ have at least two vertices (one edge) in common. When the maximum degree of the host tree T is at most h, the EPT representation of G is called an (h,2,2)-representation of G. The class of graphs which admit an (h,2,2)-representation is denoted by [h,2,2].

Notice that the class of EPT graphs is the union of the classes [h,2,2] for $h \geq 2$. It was proved that the recognition of EPT graphs is an NP-complete problem.

The EPT graphs are used in network applications, where the problem of scheduling undirected calls in a tree network is equivalent to the problem of coloring an EPT graph.

In this paper, we examine the local structure of paths passing through a given vertex of a host tree which has maximum degree h, and show these locally EPT graphs are equivalent to the line graphs of certain graphs.

Definition: Let $\langle \mathcal{P}, T \rangle$ be an EPT representation of a graph G. A pie of size n is a star subgraph of T with central vertex q and neighbors q_1,...,q_n such that each "slice" $q_i\, q\, q_{i+1}$ for $1 \leq i \leq n$ is contained in a different member of $\mathcal{P}$; addition is assumed to be module n.

Let $\langle \mathcal{P}, T \rangle$ be an EPT representation of a graph G. It was proved that if G contains a chordless cycle of length $n \geq 4$, then $\langle \mathcal{P}, T \rangle$ contains a pie of size n.

Definition: We say that $\langle \mathcal{P}, T \rangle$ is a local EPT representation of G if it is an EPT representation where all the paths of $\mathcal{P}$ share a common vertex of T.

We call G a local EPT graph if it has a local EPT representation.

Let $h \geq 5$, we say that G belongs to the class [h,2,2] local if and only if G has a local EPT representation in a host tree T with maximum degree h.

In this work, we characterize the graphs which belongs to the class [h,2,2] local.

Definition: Let G be a connected graph. We say that $v \in V(G)$ is a cut vertex of G if G-v has at least two connected components.

Theorem 1: Let $h \geq 5$. If $G \in [h, 2, 2]$ local and $G \notin [h-1, 2, 2]$ then G has no cut vertices.

We show that this special subclass of EPT graphs is equivalent to the class of line graphs of certain graphs which have certain properties.

Definition: Let H be a graph, the line graph of H, noted by L(H), has vertices corresponding to the edges of H with two vertices adjacent in L(H) if their corresponding edges of H share an endpoint.

Definition: We say that two vertices u,v of G are adjacent dominated vertices if $uv \in E(G)$ and $N_G(u) \subseteq N_G(v)$ or $N_G(v) \subseteq N_G(u)$.

Theorem 2: Let $h \geq 5$. $G \in [h, 2, 2]$ local, $G \notin [h-1, 2, 2]$ and G without adjacent dominated vertices if and only if G=L(H) with H a graph such that:

(i) —V(H)—=h; (ii) H has no vertices of degree 1; (iii) H is simple; (iv) H has no adjacent dominated vertices; (v) H has a cycle C_n, with $4 \leq n \leq h$; and every vertex of $H - C_n$ is in some path between two different vertices of C_n.

Conjecture: Let $h \geq 5$. If $G \in [h, 2, 2]$, $G \notin [h-1, 2, 2]$ but $G - v \in [h-1, 2, 2]$, for all $v \in V(G)$, then $G \in [h, 2, 2]$ local.

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