Covering edge multicolorings of complete graphs with monochromatic connected components

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Let K_n denote the complete graph on vertex set $V = [n]$. An r-edge multicoloring is a coloring of the edges with any number of colors in $[r]$. If every edge is colored by a constant number k of colors, we say that the r-edge multicoloring is k-regular. The connected components of the monochromatic graphs given by $G_i = (V_i, E_i)$, with $e \in E_i$ if the edge e has the color i, for $i \in [r]$, are called monochromatic connected components. For $r > k$, let $f(r, k)$ the smallest number such that, for any k-regular r-edge multicoloring of a complete graph, it is possible to cover the complete graph by $f(r, k)$ monochromatic connected components. A reformulation of an important special case of Ryser's conjecture states that $f(r, 1) = r - 1$ for all r. This conjecture is known to be true for $r \leq 5$. We prove, for $k \geq 2$, several exact values and bounds for $f(r, k)$, and a related result of independent interest, as the maximum number of vertices for a vertex minimal r-edge multicoloring of a complete graph coverable by three, but no by two, monochromatic connected components.

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