CHARACTERIZATIONS OF (k)-INTERVAL GRAPHS

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A graph G is an interval graph iff its vertices can be put in a one to one correspondence with a family of intervals on the real line such that two vertices in G are adjacent if and only if their corresponding intervals intersect. Such a family of intervals is called an interval model for G [1, 2].

If there is an interval model for graph G such that the intervals are of unit length, then it is called a *unit interval model* and G is a *unit interval graph*. A theorem was proved which establishes that G is a unit interval graph if and only if G is an interval graph containing no induced claw, that is, $K_{1,3}$ [6, 7].

A unit interval graph is an interval graph which admits a representation where all elements have the same length. It is known that a model with intervals of the same length and integer extremes can be built in $\mathcal{O}(n + m)$ time to represent a given unit interval graph [4, 5]. Let $k \in \mathbb{N}_0$, we define G as a (k)-interval graph iff G is a unit interval graph which admits a model of open intervals of length k and integer extremes. Likewise, we define the $[k]$-interval graphs as those unit interval graphs which admit a model with the same characteristics but with closed intervals.

In this work, we present a structural characterization of the simplicial vertices in a unit interval graph without twins, which leads to a characterization of the (k)-interval graphs. We study the structure of these graphs, finding forbidden induced subgraphs, thus a theorem fully characterising this class is posed, finding the least k such that G is a (k)-interval graph.

Moreover, the (k)-interval graphs will be studied as induced subgraphs of power of paths, continuing with the results obtained by Lin, Rautenbach, Soulignac and Szwarcfiter [3].

Furthermore, an equivalence between the $[k]$-interval and the (k)-interval graphs will be set, which allows us to use the obtained results for open intervals as well as closed intervals.

Finally, we will present open problems we are studying at the moment.

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