Counting the number of components of random real hypersurfaces

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To determine the average number of real zeroes of a univariate polynomial whose coefficients are random variables is a classical and well studied problem. A natural way to generalize it is to ask for the average number of connected components of the zero set of a random polynomial in several variables. This approach is much influenced by a random approach to Hilbert's Sixteenth Problem, to study the number and the arrangement in the projective space of the components of a real algebraic hypersurface.
The answer to the above question (both in the univariate and the multivariable case) strongly depends on the choice of the probability distribution.
In this talk I will show that, if the probability distribution is Gaussian and has no preferred points or directions in the projective space, the case of several variables can be reduced to the classical univariate problem. More precisely, the number of connected components of a random hypersurface in $\mathbb{R}P^n$ of degree d has the same order of the number of points of intersection of this hypersurface with a fixed projective line, raised to the n-th power.

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