Recent progress on boundary effects in kernel approximation

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The existence of a boundary in kernel approximation is known to drive down rates of convergence in many cases, using kernels with centers restricted to a bounded domain causes approximation orders to be saturated at a low rate (roughly half the rate of the corresponding boundary-free rates). Another unpleasant eect is that although alternative bases like Lagrange and local Lagrange functions are known to be stable and well-localized in boundary-free kernel approximation, the proof of this fact does not generalize to re- gions with boundary. Indeed, there is strong evidence that in the presence of a boundary, Lagrange functions decay at a rate that is too slow to be useful. In this talk we present recent advances in kernel approximation that treat both of the aforementioned boundary eects. We begin by discussing approximation results that over- come the low saturation order imposed by the boundary. This is an expansion from the previously understood case of surface splines on Euclidean regions to more general kernels acting on Riemannian manifolds. We follow this by presenting local basis constructions for bounded regions which yield Lp-stability results and Bernstein inequalities.