α-Molecules: Wavelets, Shearlets, and beyond

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The area of applied harmonic analysis provides a variety of multiscale systems such as wavelets, curvelets, shearlets, or ridgelets. A distinct property of each of those systems is the fact that it sparsely approximates a particular class of functions. Some of these systems even share similar approximation properties such as curvelets and shearlets which both optimally sparsely approximate functions governed by curvilinear features, a fact that is usually proven on a case-by-case basis for each different construction. The recently developed framework of parabolic molecules, which includes all known anisotropic frame constructions based on parabolic scaling, provides a unified concept for a sparse approximation results of such systems. In this talk we will introduce the novel concept of α-molecules which allows for a unified framework encompassing most multiscale systems from the area of applied harmonic analysis with the parameter α serving as a measure for the degree of anisotropy. The main result essentially states that the cross-Gramian of two systems with the same degree of anisotropy exhibits a strong off-diagonal decay. One main consequence we will discuss is that all such systems then share similar approximation properties, and desirable approximation properties of one can be deduced for virtually any other system with the same degree of anisotropy.

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