Algebraic Morse-Forman-Conley theory

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In late 90' R. Forman introduced the concept of a combinatorial vector field on a CW complex and presented a version of Morse theory for acyclic combinatorial vector fields. He also studied combinatorial vector fields without acyclicity assumption, introduced the concept of a chain recurrent set and proved Morse inequalities in this setting.

In years 2005-06 the discrete Morse theory of Forman has been generalized by several authors from the case of CW complexes to the purely algebraic case of chain complexes with a distinguished basis.

In this talk we present the Morse-Conley theory in such a purely algebraic setting. This, in particular, generalizes the definition of an isolated invariant set and its Conley index proposed recently by T. Kaczynski, M. Mrozek and Th. Wanner for simplicial complexes. Moreover, we define attractors, repellers, attractor repeller-pairs, and Morse decompositions and extend to this combinatorial/algebraic setting some classical results of Morse-Conley theory for flows.