Integral Geometry and Phase Transitions in Conic Optimization

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Integral geometry and geometric probability, going back to the work of Blaschke and Santaló, deal with measures on spaces of geometric objects, and can answer questions about the probability that random geometric objects intersect. This talk discusses various applications of (spherical) integral geometry in optimization: from the complexity theory of conic optimization to the analysis of convex approaches to solving inverse problems. In particular, it is shown how integral geometry naturally gives rise to a complete explanation of phase transition phenomena for the applicability of convex regularization to data recovery problems. We also propose extensions of the theory that allow to precisely study the probability distribution of singular values and condition numbers of conically restricted operators, and discuss relations to approaches based on geometric functional analysis.

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