Nonnegative polynomials and sums of squares on real projective varieties.

Mauricio Velasco
Universidad de los Andes, Colombia
mvelasco@uniandes.edu.co

A fundamental problem in polynomial optimization is the determination of the set of polynomials of degree d in n variables which are nonnegative on a given set $X \subseteq \mathbb{R}^n$. When X is basic semialgebraic, general certificates of positivity are available by the Positivestellensatz. In this talk we focus in the case when X is a real projective variety. The main contribution is that, for some varieties, we are able to give sharp bounds on the degree of the certificate. These bounds depend only on the classical algebro-geometric invariants of X.
Specifically, in the talk I will discuss the following two cases,
(1) The classification of all varieties X and integers d for which every polynomial of degree d which is nonnegative on X can be written as a sum of squares.
(2) Positivity certificates for curves. Let $X \subseteq \mathbb{P}^n$ be a totally real curve of arithmetic genus g whose real connected components are one-dimensional. If p is a nonnegative polynomial of degree $2s$ on X and

$$k \geq \max\left(\deg(X) - n + 1, \left\lfloor \frac{2g-1}{\deg(X)} \right\rfloor + 1\right)$$

then there exists a multiplier q of degree $2k$ such that pq is a sum of squares.

Joint work with Gregory Blekherman (Georgia Tech, USA) and Gregory G. Smith (Queen's University, Canada).