Post-Lie algebras in differential geometry and applications

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Algebraic combinatorics has become a powerful tool in the study of geometric properties of differential equations, with applications in diverse areas such as control theory, stochastic differential equations, geometric numerical integration algorithms and renormalisation theory.
Pre-Lie algebras describe algebras of flat and torsion-free connections on (mostly) euclidean spaces (Vinberg 1963). This is the algebraic foundation of Butchers B-series (Butcher 1963-72) and is closely related to a Hopf algebra appearing in renormalisation theory and non-commutative geometry (Connes-Kreimer 1999). Pre-Lie structures also appear in algebraic deformation theory (Gerstenhaber 1963). Unfortunately, pre-Lie connections do only exist on very special Lie groups, such as Euclidean spaces and certain nil-potent groups, and they are therefore not applicable for the analysis of many geometric structures appearing in mechanics and gauge field theories (principal bundles).
Post-Lie algebras is a recent invention from the last decade. The differential geometric view is the algebra of a flat connection with constant torsion. This view, with applications to numerical analysis, has been explored in a series of papers by our research group (MK-Wright 2006), MK-Lundervold 2013, Lundervold-EbrahimiFard-MK 2014). The same algebraic structure (and the name post-Lie) also appears in operad theory (Vallette 2007), where it arises as a Koszul dual of a commutative trialgebra.
Post-Lie algebras is a powerful algebraic abstraction which encodes both infinitesimal and finite aspects of flows (vector fields and their analytical or numerical flows) as well as geometric aspects such as parallel transport and curvature. There are natural post-Lie structures associated with any Lie group and more generally with homogeneous spaces and Klein geometries, where the post-Lie structure describes a connection on the Atiyah Lie algebroid.
In the talk we will survey recent developments in this field, focus on some applications in numerical integration and point out some important open research areas.

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