Geometric Numerical Integration and Computational Geometric Mechanics

Melvin Leok

University of California, San Diego, USA

mleok@math.ucsd.edu

Symmetry, and the study of invariant and equivariant objects, is a deep and unifying principle underlying a variety of mathematical fields. In particular, geometric mechanics is characterized by the application of symmetry and differential geometric techniques to Lagrangian and Hamiltonian mechanics, and geometric integration is concerned with the construction of numerical methods with geometric invariant and equivariant properties. Computational geometric mechanics blends these fields, and uses a self-consistent discretization of geometry and mechanics to systematically construct geometric structure-preserving numerical schemes.

In this talk, we will introduce a systematic method of constructing geometric integrators based on a discrete Hamilton's variational principle. This involves the construction of discrete Lagrangians that approximate Jacobi's solution to the Hamilton-Jacobi equation. Jacobi's solution can be characterized either in terms of a boundary-value problem or variationally, and these lead to shooting-based variational integrators and Galerkin variational integrators, respectively. We prove that the resulting variational integrator is order-optimal, and when spectral basis elements are used in the Galerkin formulation, one obtains geometrically convergent variational integrators.

We will also introduce the notion of a boundary Lagrangian, which is analogue of Jacobi's solution in the setting of Lagrangian PDEs. This provides the basis for developing a theory of variational error analysis for multisymplectic discretizations of Lagrangian PDEs. Equivariant approximation spaces will play an important role in the construction of geometric integrators that exhibit multimomentum conservation properties, and we will describe two approaches based on spacetime generalizations of Finite-Element Exterior Calculus, and Geodesic Finite Elements on the space of Lorentzian metrics.