Discrete inequalities for central-difference type operators

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One advantage of the energy-preserving methods is that sometimes the energy gives an a priori estimate for the (numerical) solution. For example, in the cubic nonlinear Schroedinger equation, the quartic energy function (the Hamiltonian) yields an estimate $\|u\|_\infty < \infty$ for all $t > 0$, with the aid of the discrete Gagliardo–Nirenberg and Sobolev inequalities.
Although such discrete inequalities have been known for the simplest forward (i.e. one-sided) finite difference operator, it remained open for more general operators including the standard central-difference operator, as far as the authors know. Accordingly, the analyses of energy-preserving methods with such operators remained open as well.
Recently, we found a unified way of establishing discrete inequalities for a certain range of central-difference type operators. In this talk, we show some results, and illustrate them through applications to some structure-preserving numerical schemes.

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