Structure preserving integration of hybrid dynamical systems and optimal control

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The optimal control of human walking movements requires simulation techniques, which handle the contact's establishing and releasing between the foot and the ground. The system's dynamics switches non-smoothly between phases with and without contact making the system hybrid.

During motion phases without switch, the direct transcription method Discrete Mechanics and Optimal Control (DMOCC) is used to transform the optimal control problem into a constrained optimisation problem. It involves a mechanical integrator based on a discrete constrained version of the Lagrange-d'Alembert principle. This integrator represents exactly the behaviour of the analytical solution concerning the consistency of momentum maps and symplecticity. To guarantee the structure preservation and the geometrical correctness during the establishing and releasing of contacts, the non-smooth problem is solved including the computation of the contact or contact release configuration as well as the contact time and force, instead of relying on a smooth approximation of the contact problem via a penalty potential.

While in a first approach, the sequence (not the switching time) in which the closing and opening of contacts follow each other is considered as known, a more general approach is the optimisation of the whole locomotion requiring a combined model including transitions between the different dynamical systems. Integer valued functions can be used to control if and when the switch to another dynamical system occurs, i.e. they permit to control the sequence and switching times of the dynamical systems. A variable time transformation allows to eliminate the integer valued functions and therefore to apply gradient based optimisation methods to approximate the mixed integer optimal control problem.

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