

THE BUTCHER GROUP IS A LIE GROUP

Geir Bogfjellmo

Norwegian University of Science and Technology, Norway

geir.bogfjellmo@math.ntnu.no

The concept of B-series, formal expansions of numerical methods for ordinary differential equations, has been an important tool for numerical analysts over the last decades.

In 1972, Butcher showed that numerical integrators which allow a B-series expansion, form an infinite-dimensional group under composition.

In 1998, the Butcher group was rediscovered by Connes and Kreimer in the context of renormalization in Quantum Field Theory. Connes and Kreimer also showed that, algebraically, the Butcher group is associated with a Lie algebra.

We show that the Butcher group is a Lie group modeled on a Fréchet space. The Lie algebra of Connes and Kreimer reappears as a dense subalgebra of the tangent space at the identity of this group.

We explore the properties of the Butcher group from the point of infinite dimensional Lie group, thus complementing the algebraic treatment by Connes and Kreimer.

Joint work with Alexander Schmeding (Norwegian University of Science and Technology).