High-dimensional algorithms in weighted Hermite spaces of analytic functions

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We consider integration and approximation of functions in a class of Hilbert spaces of analytic functions defined on the $\mathbb{R}^s$. The functions are characterized by the property that their Hermite coefficients decay exponentially fast. The function spaces are weighted by two weight sequences. For numerical integration, we use Gauss-Hermite quadrature rules and show that the errors of our algorithms decay exponentially fast. Furthermore, we consider L_2-approximation where the algorithms use information based on either arbitrary linear functionals or function evaluations. Also in the case of approximation we obtain exponential error convergence. For given $\varepsilon > 0$, we study tractability in terms of s and $\log \varepsilon^{-1}$ and give necessary and sufficient conditions under which we achieve exponential convergence with various types of tractability.

Joint work with Christian Irrgeher (Johannes Kepler University Linz, Austria), Gunther Leobacher (Johannes Kepler University Linz, Austria), Friedrich Pillichshammer (Johannes Kepler University Linz, Austria) and Henryk Wozniakowski (Columbia University, USA; University of Warsaw, Poland).