The ANOVA decomposition of a non-smooth function of an infinite number of variables

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In this work we extend our earlier work motivated by path-dependent option pricing problems, in which we tried to understand how it is that sparse grid and QMC methods can be applied successfully to option pricing problems, even though the integrands do not live in any mixed derivative smoothness class. That difficulty derives from the "max function" in the integrand, describing the fact that options are considered worthless if the payoff falls below the strike price.

In a previous paper (Math. Comp. 82, 383-400, 2013) we showed that if the expected value is expressed as an integral over $\mathbb{R}^d$ then the classical ANOVA decomposition of the integrand for an arithmetic Asian option can have every term smooth except for the very highest term. That highest ANOVA term has many discontinuities in first partial derivatives, but in most cases is expected to be pointwise very small. In the present work we consider the ANOVA decomposition of the corresponding continuous problem in the Brownian bridge (or Levy-Ciesielski) formulation, and show that in this case **every term in the (infinite) ANOVA decomposition is smooth.** With this result we are preparing for an error analysis of the cubature problem for option pricing problem, in which the discrete-time problem is approximated by the continuous problem, and the error analysis then applied to the truncated infinite ANOVA expansion, in which every term is smooth.

Joint work with Frances Kuo (UNSW), Michael Griebel (Bonn).