Preasymptotic estimates for approximation of multivariate Sobolev functions

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The talk is concerned with optimal linear approximation of functions in isotropic periodic Sobolev spaces $H^s(\mathbb{T}^d)$ of fractional smoothness $s > 0$ on the d-dimensional torus, where the error is measured in the L_2-norm. The asymptotic rate – up to multiplicative constants – of the approximation numbers is well known. For any fixed dimension $d \in \mathbb{N}$ and smoothness $s > 0$ one has

$$(\star) \qquad a_n(I_d : H^s(\mathbb{T}^d) \to L_2(\mathbb{T}^d)) \sim n^{-s/d} \qquad \text{as} \quad n \to \infty\,.$$

In the language of IBC, the n-th approximation number $a_n(I_d)$ is nothing but the worst-case error of linear algorithms that use at most n arbitrary linear informations. Clearly, for numerical issues and questions of tractability one needs precise information on the constants that are hidden in $(\star)$, in particular their dependence on d.

For any fixed smoothness $s > 0$, the exact asymptotic behavior of the constants as $d \to \infty$ will be given in the talk. Moreover, I will present sharp two-sided estimates in the preasymptotic range, that means for 'small' n. Hereby an interesting connection to entropy numbers in finite-dimensional ℓ_p-spaces turns out to be very useful.

Joint work with Sebastian Mayer (Bonn), Winfried Sickel (Jena) and Tino Ullrich (Bonn).