Numerical integration of functions with mixed smoothness

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We consider numerical integration of d-dimensional functions in (Besov and Triebel-Lizorkin) spaces $A^s_{p,\theta}$ of dominating mixed smoothness on the unit cube. For functions with zero boundary condition we prove that a (equally weighted) cubature rule, as proposed by Frolov in 1976, has the optimal order of convergence if $p, \theta \in (0, \infty)$ and $s > \max\{1/p, 1/\theta\}$, where s is the smoothness parameter. In particular, this implies the optimal order of convergence in Sobolev spaces of mixed smoothness in the same range of the parameters, which generalizes the previously known results to $p < 2$ and non-integer smoothness. In the region of "small smoothness", $s \in (1/p, 1/\theta]$, we prove an upper bound that is expected to be optimal. This bound for Triebel-Lizorkin spaces was only known for $d = 2$. By standard modifications of the algorithm we obtain the same results for functions without boundary conditions.

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