Finite N corrections to the Tracy-Widom distribution at the hard edge of the Laguerre-Wishart ensemble of complex random matrices

Grégory Schehr
University of Orsay-Paris Sud, France
gregory.schehr@u-psud.fr

We study the probability distribution function (PDF) of the smallest eigenvalue of Laguerre-Wishart matrices $W = X^\dagger X$ where X is a random $M \times N$ ($M \geq N$) matrix, with complex Gaussian independent entries. We compute this PDF in terms of semi-classical orthogonal polynomials, which can be viewed as a deformation of Laguerre polynomials. By analyzing these polynomials, and their associated recurrence relations, in the limit of large N, large M with $M/N \to 1$ – i.e. for quasi-square large matrices X – we show that this PDF can be expressed in terms of the solution of a Painlevé III equation, as found by Tracy and Widom by analyzing a Fredholm determinant built from the Bessel kernel. In addition, our method allows us to compute the first $1/N$ corrections to this limiting Tracy-Widom distribution (at the hard edge). Our computations corroborate a recent conjecture by Edelman, Guionnet and Péché.

Joint work with Anthony Perret (University of Orsay, Paris-Sud).