Computing the parameterized differential Galois group of a second-order linear differential equation with parameters

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Consider a linear differential equation

$$\frac{\partial^2 Y}{\partial x^2} + r_1 \frac{\partial Y}{\partial x} + r_0 Y = 0,$$

where the coefficients $r_1, r_0 \in \mathbb{C}(t_1, \ldots, t_m, x)$. The parameterized Picard-Vessiot theory developed by Phyllis Cassidy and Michael Singer associates a differential Galois group G to such an equation. In analogy with the classical Picard-Vessiot theory of Kolchin, G measures the differential-algebraic relations amongst the solutions to the equation, with respect to $\frac{\partial}{\partial x}$ as well as $\frac{\partial}{\partial t_1}, \ldots, \frac{\partial}{\partial t_m}$.
Relying on earlier work by Thomas Dreyfus, I will describe a complete set of algorithms to compute G, and how these algorithms lead to a simple procedure to decide whether any of the solutions to the equation are differentially transcendental with respect to one or several of the parametric derivations $\frac{\partial}{\partial t_1}, \ldots, \frac{\partial}{\partial t_m}$.