A decision method for integrability of partial differential algebraic Pfaffian systems.

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Let $m, n \in \mathbb{N}$. Let $x_1, \ldots, x_m$ be independent variables and $\mathbf{y} := y_1, \ldots, y_n$ be differential unknowns. For each pair (i, j), $1 \leq i \leq n$, $1 \leq j \leq m$, let f_{ij} be a polynomial in $\mathbb{C}[\mathbf{y}]$. A differential algebraic Pfaffian system is a system of differential equations as follows:

$$\Sigma = \begin{cases} \dfrac{\partial y_i}{\partial x_j} \& = \& f_{ij}(\mathbf{y}), \& \quad \text{for } i = 1, \ldots, n \text{ and } j = 1 \ldots m, \\ \mathbf{g}(\mathbf{y}) \& = \& 0 \& \end{cases}$$

where $\mathbf{g} := g_1, \ldots, g_s$ are polynomials in $\mathbb{C}[\mathbf{y}]$.

In this work we are interested in the integrability of these systems, that is, in the existence of infinitely differentiable functions over an open set $\mathcal{U}$ of $\mathbb{C}^m$ that are solutions of Σ. The classical Frobenius Theorem (1877) establishes conditions for a Pfaffian system, without algebraic constraints, to be completely integrable. We focus on the integrability, not necessarily complete, of systems like Σ.

We associate to each system Σ a strictly decreasing chain of algebraic varieties in $\mathbb{C}^n$ of length at most $n + 1$. We prove that a necessary and sufficient condition for the existence of solutions for Σ is that the smallest variety of this chain is nonempty. From this result, we are able to show an effective procedure that allows us to decide whether a Pfaffian system is integrable in triple exponential time in n, the number of unknowns.

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