Generalized barycentric coordinates and algebraic geometry

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Let P_d be a convex polygon with d vertices. The associated Wachspress surface W_d is a fundamental object in approximation theory, defined as the image of the rational map w_d from $\mathbb{P}^2$ to $\mathbb{P}^{d-1}$, determined by the Wachspress barycentric coordinates for P_d. We show w_d is a regular map on a blowup X_d of $\mathbb{P}^2$, and if $d > 4$ is given by a very ample divisor on X_d, so has a smooth image W_d. We determine generators for the ideal of W_d, and prove that in graded lex order, the initial ideal of $I(W_d)$ is given by a Stanley-Reisner ideal. As a consequence, we show that the associated surface is arithmetically Cohen-Macaulay, of Castelnuovo-Mumford regularity two, and determine all the graded betti numbers of $I(W_d)$.

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