Finite element spectral approximation of the curl operator in multiply connected domains

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A couple of numerical methods based on Nedelec finite elements have been recently introduced and analyzed in [1] to solve the eigenvalue problem for the curl operator in simply connected domains. This topological assumption is not just a technicality, since the eigenvalue problem is ill-posed on multiply connected domains, in the sense that its spectrum is the whole complex plane as is shown in [2]. However, additional constraints can be added in order to recover a well posed problem with a discrete spectrum [2,3]. We choose as additional constraint a zero-flux condition of the curl on all the cutting surfaces. We introduce two weak formulations of the corresponding problem, which are convenient variations of those studied in [1]; one of them is mixed and the other a Maxwell-like formulation. We prove that both are well posed and show how to modify the finite element discretization from [1] to take care of these additional constraints. We prove spectral convergence of both discretizations and establish a priori error estimates. We also report numerical tests which allow assessing the performance of the proposed methods.

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