Efficient minimax strategies for online prediction

Peter Bartlett

UC Berkeley and Queensland University of Technology, USA

peter@berkeley.edu

Consider prediction games in which, in each round, a strategy makes a decision, then observes an outcome and pays a loss. The aim is to minimize the regret, which is the amount by which the total loss incurred exceeds the total loss of the best decision in hindsight. We study the case where decisions and outcomes lie in a convex subset of a Hilbert space, and loss is squared distance. When the set is the simplex, this is the 'Brier game,' studied for the calibration of sequential probability forecasts; when it is the Euclidean ball, the game is related to sequential Gaussian density estimation. We show that the value of the game depends only on the radius of the smallest ball that contains the convex subset, and that the minimax optimal strategy is a simple shrinkage strategy that can be efficiently computed, given the center of the smallest ball.

Joint work with Wouter Koolen (UC Berkeley and Queensland University of Technology) and Alan Malek (UC Berkeley).