Orthogonal and para-orthogonal polynomials on the unit circle

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When a nontrivial measure μ on the unit circle satisfies the symmetry $d\mu(e^{i(2\pi-\theta)}) = -d\mu(e^{i\theta})$ then the associated orthogonal polynomials on the unit circle, say S_n, are all real. In this case, in [3], Delsarte and Genin have shown that the two sequences of para-orthogonal polynomials $\{zS_n(z) + S_n^*(z)\}$ and $\{zS_n(z) - S_n^*(z)\}$ satisfy three term recurrence formulas and have explored some further consequences of these sequences of polynomials such as their connections to sequences of orthogonal polynomials on the interval $[-1, 1]$. Even though results presented in Delsare and Genin [4] extend these partly to include any nontrivial measures on the unit circle, only recently, in [2] (and also [1]), the extension associated with the para-orthogonals polynomials $zS_n(z) - S_n^*(z)$ was studied extensively. The results given in [2], especially from the point of view of three term recurrence, provide also as a nice application a characterization for any pure points in the measure. The main objective of the present contribution is to provide some recent developments concerning the extension for the para-orthogonals polynomials $zS_n(z) + S_n^*(z)$ to cover all nontrivial measures on the unit circle.

References

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Joint work with Cleonice F. Bracciali (Universidade Estadual Paulista, Brazil) and A. Swaminathan (IIT Roorkee, India).