Quasi-orthogonality of some ${}_pF_q$ hypergeometric polynomials

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We prove the quasi-orthogonality of some general classes of hypergeometric polynomials of the form

$$
{}_pF_q\left(\begin{array}{c} -n, \beta_1+k, \alpha_3 \ldots, \alpha_p \\ \beta_1, \ldots, \beta_q \end{array}; x\right) = \sum_{m=0}^{n} \frac{(-n)_m(\beta_1+k)_m(\alpha_3)_m \ldots (\alpha_p)_m}{(\beta_1)_m \ldots (\beta_q)_m} \frac{x^m}{m!}
$$

for $k \in \{1, 2, \ldots, n-1\}$ which do not appear in the Askey scheme for hypergeometric orthogonal polynomials. Our results include, as a special case, the order one quasi-orthogonal Sister Celine polynomials

$$
f_n(a,x) = {}_3F_2\left(\begin{array}{c} -n, n+1, a \\ \frac{1}{2}, 1 \end{array}; x\right) = \sum_{m=0}^{n} \frac{(-n)_m(n+1)_m(a)_m}{\left(\frac{1}{2}\right)_m (1)_m} \frac{x^m}{m!}
$$

with $a = 2$ and $a = 3/2$ considered by Dickenson in 1961. The location and interlacing of the real zeros of the quasi-orthogonal polynomials are also discussed.

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