Extending Askey tableau by the inclusion of Krall and exceptional polynomials

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Krall and exceptional polynomials are two of the more important extensions of the classical families of Hermite, Laguerre and Jacobi. On the one hand, Krall or bispectral polynomials are orthogonal polynomials which are also eigenfunctions of a differential operator of order bigger than two (and polynomial coefficients). The first examples were introduced by H. Krall in 1940, and since the eighties a lot of effort has been devoted to this issue (with contributions by L.L. Littlejohn, A.M. Krall, J. and R. Koekoek. A. Grünbaum and L. Haine (and collaborators), K.H. Kwon (and collaborators), A. Zhedanov, P. Iliev, and many others). On the other hand, exceptional polynomials are orthogonal polynomials which are also eigenfunctions of a second order differential operator, but they differ from the classical polynomials in that their degree sequence contains a finite number of gaps, and hence the differential operator can have rational coefficients. In mathematical physics, these functions allow to write exact solutions to rational extensions of classical quantum potentials. Exceptional polynomials appeared some seven years ago, but there has been a remarkable activity around them mainly by theoretical physicists (with contributions by D. Gómez-Ullate, N. Kamran and R. Milson, Y. Grandati, C. Quesne, S. Odake and R. Sasaki, and many others). Taking into account these definitions, it is scarcely surprising that no connection has been found between Krall and exceptional polynomials. However, if one considers difference operators instead of differential ones (that is, the discrete level of Askey tableau), something very exciting happens: Duality (i.e., swapping the variable with the index) interchanges Krall discrete and exceptional discrete polynomials. This unexpected connection of Krall discrete and exceptional polynomials allows a nice and important extension of Askey tableu. Also, this worthy fact can be used to solve some of the most interesting questions concerning exceptional polynomials; for instance, to find necessary and sufficient conditions such that the associated second order differential operators do not have any singularity in their domain.