Completed Stieltjes interlacing of zeros of different orthogonal polynomials

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Consider the orthogonal sequence $\{p_n\}_{n=0}^{\infty}$. If the polynomials p_n and p_{n-m}, $m = 2, 3, \ldots, n-1$, have no common zeros, there exists a real polynomial of degree $m-1$, completely determined by the coefficients in the three term recurrence relation satisfied by the orthogonal sequence $\{p_n\}_{n=0}^{\infty}$, whose real simple zeros provide a set of points that, together with the zeros of p_{n-m}, completely interlace with the zeros of p_n, a property we refer to as completed Stieltjes interlacing. The conditions under which completed Stieltjes interlacing holds between the zeros of polynomials from different orthogonal sequences are studied and this leads to a set of points that can be applied as bounds for the extreme zeros of the polynomials. We apply our results to some discrete orthogonal polynomials and identify new bounds for the extreme zeros of these polynomials.

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