Bivariate Lagrange interpolation and quadrature formulas at the node sets of Lissajous curves

Wolfgang Erb
University of Lübeck, Germany
erb@math.uni-luebeck.de

Motivated by an application in Magnetic Particle Imaging, we study bivariate Lagrange interpolation at the node points of Lissajous curves. The resulting theory is a generalization of the polynomial interpolation theory developed for a node set known as Padua points. With appropriately defined polynomial spaces, we show that the node points of Lissajous curves allow unique interpolation. Further, these node sets can be used as sampling points for quadrature rules for integrals with product Chebyshev weights. An explicit formula for the Lagrange polynomials allows to compute the interpolating polynomial with a simple algorithmic scheme. Compared to the already established schemes of the Padua and Xu points, the numerical results for the proposed scheme show similar approximation errors and a similar growth of the Lebesgue constant.

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