Orthogonal Rational Functions and non-stationary stochastic processes

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We present a generalization of the Szegö theory of orthogonal polynomials on the unit circle to orthogonal rational functions. Just like the Kolmogorov-Krein-Szegö theorem may be interpreted as an asymptotic estimate of the prediction error for stationary stochastic processes, the present theory yields an asymptotic estimate of the prediction error for certain, possibly non-stationary, stochastic processes. The latter admit a spectral calculus where the time-shift corresponds to multiplication by elementary Blaschke products of degree 1 (that reduce to multiplication by the independent variable in the stationnary case). When the poles of the best predictor tend to a point on the unit circle where the spectral density is non-zero, the prediction error goes to zero, that is, the process is asymptotically deterministic.

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