Multilevel Monte-Carlo Methods for hyperbolic PDEs with random input data

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We consider random scalar hyperbolic conservation laws (RSCLs) in spatial dimension d with bounded random flux functions which are P-a.s. Lipschitz continuous with respect to the state variable.
There exists a unique random entropy solution (i.e., a strongly measurable mapping from a probability space into $C([0,T];L^1(\mathbb{R}^d))$) with finite second moments.
We present a convergence analysis of a Multi-Level Monte-Carlo Front-Tracking (MLMCFT) algorithm. It is based on "pathwise" application of the Front-Tracking Method for deterministic SCLs.
We compare the MLMCFT algorithms to the Multi-Level Monte-Carlo Finite-Volume methods. Due to the absence of a CFL time step restriction in the pathwise front tracking scheme, we can prove favourable complexity estimates: in spatial dimension $d \geq 2$, the mean field of the random entropy solution can be approximated numerically with (up to logarithmic terms) the same complexity as the solution of one instance of the deterministic problem, on the same mesh.
We then present results on large scale simulations of MLMC for wave propagation in heterogeneous media with log-Gaussian random coefficients. Here, conventional explicit timestepping schemes encounter the CFL constraint which, due to the lognormal Gaussian constitutive parameter, is random. A novel probabilistic complexity analysis and and adaptive load balancing algorithm achieve near linear strong scaling on up to 40K processors.

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