Higher order QMC Galerkin Discretization for parametric operator equations

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We discuss quasi-Monte Carlo methods to approximate the expected values of linear functionals of Petrov-Galerkin discretizations of parametric operator equations which depend on a possibly infinite sequence of parameters. Such problems arise in the numerical solution of differential and integral equations with random field inputs. We analyze the regularity of the solutions with respect to the parameters in terms of the rate of decay of the fluctuations of the input field. If $p \in (0,1]$ denotes the “summability exponent” corresponding to the fluctuations in affine-parametric families of operators, then we prove that deterministic “interlaced polynomial lattice rules” of order $\alpha = \lfloor 1/p \rfloor + 1$ in s dimensions with N points can be constructed using a fast component-by-component algorithm, in $\mathcal{O}(\alpha\, s\, N \log N + \alpha^2\, s^2 N)$ operations, to achieve a convergence rate of $\mathcal{O}(N^{-1/p})$, with the implied constant independent of s. This dimension-independent convergence rate is superior to the rate $\mathcal{O}(N^{-1/p+1/2})$ for $2/3 \leq p \leq 1$, which was recently established for randomly shifted lattice rules under comparable assumptions. In our analysis we use a non-standard Banach space setting and introduce “smoothness-driven product and order dependent (SPOD)” weights for which we develop a new fast CBC construction.

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