How to simulate stochastic differential equations without discretizing time

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Numerical methods for SDEs are primarily based on pathwise approximation, basically if the Brownian motion is given in advance then the solution to the SDE (in a pathwise sense) looks like the solution to an ODE. Since the ODE solution is unique and smooth, it makes sense to discretize in time. In fact, solving the ODE is quite often thought of as an interpolation problem, where the largest time step is used to fit the approximation to the solution. The situation in the SDE context is a bit different: the solution to the SDE is continuous, but not differentiable in general, and an ensemble of trajectories originates from any initial point. Passing to the continuous limit is also nontrivial in the SDE context because existence of a pathwise unique solution imposes stringent conditions on the coefficients and domain of the SDE, which can be difficult to satisfy, and in fact, are not needed in practice. In actual computations one mainly considers the approximation of statistics of the law of the solution (mean first passage times, multi-time expectations, invariant measures, etc.), which is the main aim of SDE solvers, and the existence of a pathwise unique solution is not necessary for convergence in law. In this talk I will numerically illustrate why standard integrators are sometimes inconvenient to use, and present a different approach motivated by the issues encountered when simulating SDEs coming from quantitative finance, population dynamics, chemical kinetics, epidemiology, and polymeric fluids. This talk is based on joint work with Eric Vanden-Eijnden at NYU.

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