The L-functions and modular forms database project

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The simplest and most famous L-function is the Riemann Zeta function. L-functions are ubiquitous in number theory, and have applications to mathematical physics and cryptography. Two of the seven Clay Mathematics Million Dollar Millennium Problems deal with their properties: the Riemann Hypothesis and the Birch and Swinnerton-Dyer Conjecture. They arise from and encode information about a number of mathematical objects, and also provide links between them: for example, Wiles' celebrated proof of Fermat's Last Theorem centred on proving that L-functions associated with certain elliptic curves were also associated with other objects called modular forms.

At least a dozen different mathematical objects are connected in various ways to L-functions. The study of those objects is highly specialized, and most mathematicians have only a vague idea of the objects outside their specialty and how everything is related. Helping mathematicians to understand these connections was the motivation for the L-functions and Modular Forms Database (LMFDB) project, which started at AIM in 2007 and has been supported by major grants from the NSF and (currently) the UK EPSRC. Its mission is to chart the landscape of L-functions and modular forms in a systematic and concrete fashion. This involves developing their theory, creating and improving algorithms for computing and classifying them, and hence discovering new properties of these functions, and testing fundamental conjectures.

In the lecture I will explain and demonstrate how we organise our large collection of data and display it, together with the interrelations between linked objects, through our website [www.lmfdb.org]. I will also show how this has been built via a world-wide collaborative open source project which we hope may become a model for others.