AFEM for the Laplace-Beltrami Operator: Convergence Rates

Ricardo H. Nochetto
University of Maryland, USA
rhn@math.umd.edu

Elliptic partial differential equations (PDEs) on surfaces are ubiquitous in science and engineering. We present several geometric flows governed by the Laplace-Beltrami operator. We design a new adaptive finite element method (AFEM) with arbitrary polynomial degree for such an operator on parametric surfaces, which are globally Lipschitz and piecewise in a suitable Besov class: the partitions thus match possible kinks. The idea is to have the surface sufficiently well resolved in W^1_∞ relative to the current resolution of the PDE in H^1. This gives rise to a conditional contraction property of the PDE module and yields optimal cardinality of AFEM. Moreover, we relate the approximation classes to Besov classes.

If the meshes do not match the kinks, or they are simply unknown beforehand, we end up with elliptic PDEs with discontinuous coefficients within elements. In contrast to the usual perturbation theory, we develop a new approach based on distortion of the coefficients in L_q with $q < \infty$. We then use this new distortion theory to formulate a new AFEM for such discontinuity problems, show optimality of AFEM in the sense of distortion versus number of computations, and report insightful numerical results supporting our analysis.

Joint work with A. Bonito (Texas AM University, USA), M. Cascon (Universidad de Salamanca, Spain), R. DeVore (Texas AM University, USA), K. Mekchay (Chulalongkorn University, Thailand) and P. Morin (Universidad Nacional del Litoral, Argentina).