

DIFFERENTIAL GROUPS AND THE GAMMA FUNCTION

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In 1887, Hoelder proved that the Gamma Function, defined by the difference equation $y(x+1) = x y(x)$, satisfies no nonzero polynomial differential equation with complex coefficients. In the last several years Galois theories have been developed that reprove this result and allow one to characterize when functions satisfying certain linear differential or difference equations also satisfy auxiliary difference or differential equations. These Galois theories allow one to reduce such kinds of questions to questions concerning linear differential or difference groups, that is groups of matrices whose entries are functions satisfying a fixed set of differential or difference equations. I will give an introduction to the theory of these groups and the related Galois theories and survey recent results applying these theories to questions of functional transcendence.