Zeros (of some polynomials) prefer curves

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Polynomials satisfying non-Hermitian orthogonality relations appear naturally in approximation theory (as denominators of Padé and Hermite-Padé approximants), random matrix theory, differential equations and special functions, just to mention a few branches of analysis. They are defined by orthogonality relations on the complex plane in which we can freely deform the paths of integration. It is natural to ask where the zeros of these polynomials choose to go. Computation shows that they like to align themselves along certain curves on the plane. What are these curves? In some cases we can answer this question, at least asymptotically. But the answer connects fascinating mathematical objects, such as extremal problems in electrostatics, Riemann surfaces, trajectories of quadratic differentials, algebraic functions; this list is not complete.

This is a brief survey of some ideas related to this problem, starting from the classics, with an emphasis on the breakthrough developments in the 1980-ies, and finishing with some recent results and open problems.

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