On Adaptive Multilevel Monte Carlo and Multi-Index Monte Carlo

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We provide a quick glance into recently developed Adaptive Multilevel Monte Carlo (MLMC) Methods for the following widely applied mathematical models: (i) Itô Stochastic Differential Equations, (ii) Stochastic Reaction Networks modeled by Pure Jump Markov Processes and (iii) Partial Differential Equations with random inputs. In this context, the notion of adaptivity includes several aspects such as mesh refinements based on either a priori or a posteriori error estimates, the local choice of different time stepping methods and the selection of the total number of levels and the number of samples at different levels. Our Adaptive MLMC estimator is based on a hierarchy of adaptively refined, non-uniform time discretizations, and, as such, it may be considered a generalization of the uniform discretization MLMC method introduced independently by M. Giles and S. Heinrich. In particular, we show that under some assumptions our adaptive MLMC algorithms are asymptotically accurate and have the correct complexity with an improved control of the complexity constant factor in the asymptotic analysis. Moreover, we show the asymptotic normality of the statistical error in the MLMC estimator, justifying in this way our error estimate that allows prescribing both the required accuracy and confidence level in the final result.

We will show several examples, including some dynamics with singularities and/or non-smooth observables, to illustrate the above results and the corresponding computational savings.

Finally, we will briefly describe the Multi Index Monte Carlo method, presenting new and improved complexity results which are natural generalizations of Giles's MLMC analysis.

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