Contents

Plenary talks

Plenary talk - December 11, 9:30 – 10:25

On subset sums

Endre Szemeredi
Hungarian Academy of Science and Rutgers University , Hungary and USA
szemered@cs.rutgers.edu

Let $A \subset [1; N]$ be a set of integers. We denote by S_A the collection of partial sums of A,

$$S_A = \left\{ \sum_{x \in B} x : B \subset A \right\}.$$

For a positive integer $l \leq A$ we denote by l^*A the collection of partial sums of l elements of A,

$$l^*A = \left\{ \sum_{x \in B} x : B \subset A, |B| = l \right\}.$$

We will discuss the structure of l^*A and give a tight bound of the size of A not containing an N element arithmetic progression.

Some of the results are joint with Van Vu, the others are joint work with Simao Herdade.

Plenary talk - December 11, 11:00 – 11:55

Some Problems For This Century

Michael Shub
City University of New York, USA
shub.michael@gmail.com

In 1998 Steve Smale published a list of 18 problems for the next century. Four of them arose in our joint work, one with Lenore Blum. I will speak about progress (or lack of it) on some of these problems. In particular, Problem 17 on finding roots of polynomial equations will be included with some recent results about the eigenvalue, eigenvector problem. These problems involve finding well conditioned polynomial systems with respect to roots or matrices with respect to eigenvalues, eigenvectors. For one homogeneous polynomial in two variables the problem is related to Problem 7 on the distribution of points on the 2-sphere.

Time permitting, I will speak about some ideas related to Problem 4 on integer zeros of a polynomial.

Plenary talk - December 12, 9:00 – 9:55

The L-functions and modular forms database project

John Cremona
University of Warwick, UK
j.e.cremona@warwick.ac.uk

The simplest and most famous L-function is the Riemann Zeta function. L-functions are ubiquitous in number theory, and have applications to mathematical physics and cryptography. Two of the seven Clay Mathematics Million Dollar Millennium Problems deal with their properties: the Riemann Hypothesis and the Birch and Swinnerton-Dyer Conjecture. They arise from and encode information about a number of mathematical objects, and also provide links between them: for example, Wiles' celebrated proof of Fermat's Last Theorem centred on proving that L-functions associated with certain elliptic curves were also associated with other objects called modular forms.

At least a dozen different mathematical objects are connected in various ways to L-functions. The study of those objects is highly specialized, and most mathematicians have only a vague idea of the objects outside their specialty and how everything is related. Helping mathematicians to understand these connections was the motivation for the L-functions and Modular Forms Database (LMFDB) project, which started at AIM in 2007 and has been supported by major grants from the NSF and (currently) the UK EPSRC. Its mission is to chart the landscape of L-functions and modular forms in a systematic and concrete fashion. This involves developing their theory, creating and improving algorithms for computing and classifying them, and hence discovering new properties of these functions, and testing fundamental conjectures.

In the lecture I will explain and demonstrate how we organise our large collection of data and display it, together with the interrelations between linked objects, through our website [www.lmfdb.org]. I will also show how this has been built via a world-wide collaborative open source project which we hope may become a model for others.

Plenary talk - December 12, 11:00 – 11:55

Multipliers and contraints for spline-based methods

Annalisa Buffa
Istituto di Matematica e Tecnologie Informatiche , ITALY
annalisa.buffa@imati.cnr.it

Lagrange multipliers are used to impose constraints to the solution of a partial differential equation in a variational way. Relevant examples are the continuity across interfaces, or conservation of mass and volumes. It is well known that the resulting system represents a saddle point problem and its discretization requires special care.

Indeed, the choice of discretization spaces for multipliers depend upon the discretization of the primal variables and it is key to stability. I will discuss the choices that are possible when the primal variables are described by splines or NURBS in various contexts. I will introduce mortaring techniques to impose continuity across non-matching spline surfaces and their applications to contact mechanics, as well as the numerical treatment of (quasi-)incompressibility and its application to large deformation problems.

Joint work with Pablo Antolín (University of Pavia, Italy), E. Brivadis (IUSS, Pavia, Italy) and Giancarlo Sangalli (University of Pavia, Italy).

Plenary talk - December 13, 9:00 – 9:55

Pursuit of Low-dimensional Structures in High-dimensional Data

Yi Ma
ShanghaiTech University, P. R. China
mayi@shanghaitech.edu.cn

In this talk, we will discuss a new class of models and techniques that can effectively model and extract rich low-dimensional structures in high-dimensional data such as images and videos, despite nonlinear transformation, gross corruption, or severely compressed measurements. This work leverages recent advancements in convex optimization for recovering low-rank or sparse signals that provide both strong theoretical guarantees and efficient and scalable algorithms for solving such high-dimensional combinatorial problems. These results and tools actually generalize to a large family of low-complexity structures whose associated regularizers are decomposable. We illustrate how these new mathematical models and tools could bring disruptive changes to solutions to many challenging tasks in computer vision, image processing, and pattern recognition. We will also illustrate some emerging applications of these tools to other data types such as web documents, image tags, microarray data, audio/music analysis, and graphical models.

Joint work with John Wright (Columbia University), Emmanuel Candes (Stanford University), Zhouchen Lin (Peking University) and Yasuyuki Matsushita (Microsoft Research Asia).

Plenary talk - December 13, 11:00 – 11:55

Heating the sphere

Carlos Beltrán
Universidad de Cantabria, Spain
beltranc@unican.es

Where would you allocate N sources of heat in the 2-dimensional sphere in order to maximize the steady state average temperature, assuming a constant cooling rate everywhere?

In this talk I will describe a theoretical solution to this facility location problem, relating it to other classical questions in potential theory, stability of polynomial zeros, eigenvalue computations and numerical integration. Novel results about some of these classical problems will be presented as a consequence of the analysis.

Joint work with : Different coauthors in several parts of the talk will be credited in the slides.

Plenary talk - December 15, 9:30 – 10:25

The joy and pain of skew symmetry

Arieh Iserles
University of Cambridge, United Kingdom
ai@damtp.cam.ac.uk

Skew symmetry is good for you! More specifically, once a derivative is discretised with a skew-symmetric matrix, a numerical PDE scheme displays a range of favourable features: it is stable and various structural features of the original equation are preserved.

It is easy to design skew-symmetric differentiation matrices in tandem with periodic boundary conditions, e.g. using spectral collocation, and the first part of the talk will be devoted to a detailed study of the discretisation of the semiclassical Schrödinger equation using asymptotic splittings. Our main tools are a free Lie algebra of operators, palindromic Zassenhaus splitting and the symmetric BCH formula.

In the second part of the talk we consider Dirichlet boundary conditions, and this is the instant the pain kicks in. Using finite differences on a uniform grid, the highest order of a skew-symmetric differentiation matrix is just two! We will derive detailed necessary conditions for a non-uniform grid so that a skew-symmetric differentiation matrix of given order exists and prove that they can be always realised by a banded matrix. We will also discuss the state of the art in the construction and properties of such matrices.

Plenary talk - December 15, 11:00 – 11:55

Stochastic Asynchronous Parallel Methods in Optimization

Stephen Wright
University of Wisconsin-Madison, USA
swright@cs.wisc.edu

The ubiquity of multicore computer architectures and clusters and the advent of new applications of optimization in machine learning, data analysis and other areas has prompted a reevaluation of elementary optimization methods that had long been out of fashion. Stochastic synchronous parallel variants of such methods as stochastic gradient, coordinate descent, and successive projections have been a particular focus of interest. We survey such methods here, presenting computational results for several of them. We then focus on two such methods - coordinate descent for convex optimization and the Kaczmarz method for linear systems - and introduce a model of multicore computation that is both close to reality and amenable to analysis of convergence behavior. We show in particular that there is a threshold number of cores below which near-linear speedup of the algorithm (as a function of the number of cores) can be expected.

Joint work with Ji Liu (University of Rochester).

Plenary talk - December 16, 9:00 – 9:55

Architectural Geometry

Helmut Pottmann
King Abdullah University of Science and Technology, Saudi Arabia
helmut.pottmann@kaust.edu.sa

Free forms constitute one of the major trends within contemporary architecture. While the digital design of freeform geometry with current modeling tools is well understood, the actual fabrication on the architectural scale is a big challenge: one has to decompose the skins into manufacturable panels, provide appropriate support structures, meet structural constraints and last, but not least make sure that the cost does not become excessive. These practical requirements form a rich source of research topics in geometry and geometric computing. The talk will provide an overview of recent progress in the emerging field of Architectural Geometry, elaborate on important relations to contemporary research in Discrete Differential Geometry and Geometric Optimization, and illustrate the transfer of mathematical research into the architectural practice at hand of selected projects.

Plenary talk - December 16, 11:00 – 11:55

Liberating the Dimension - Quasi Monte Carlo Methods for High Dimensional Integration

Frances Kuo
University of New South Wales, Australia
f.kuo@unsw.edu.au

High dimensional problems are coming to play an ever more important role in applications, including, for example, option pricing problems in mathematical finance, maximum likelihood problems in statistics, and porous flow problems in computational physics and uncertainty quantification. High dimensional problems pose immense challenges for practical computation, because of a nearly inevitable tendency for the cost of computation to increase exponentially with dimension. Effective and efficient methods that do not suffer from this "curse of dimensionality" are in great demand, especially since some practical problems are in fact infinite dimensional.

In this talk I will start with an introduction to "quasi-Monte Carlo methods", focusing on the theory and construction of "lattice rules" (order one) and "interlaced polynomial lattice rules" (higher order) developed in the past decade. Then I will showcase our very latest work on how this modern theory can be "tuned" for a given application. The motivating example will involve an elliptic PDE with a random coefficient, which is based on a simplified porous flow problem where the permeability is modeled as a random field.

Plenary talk - December 17, 9:00 – 9:55

On the characterization of approximation spaces in Nonlinear Approximation

Pencho Petrushev
University of South Carolina, USA
pencho@math.sc.edu

An overview of old and new results in Nonlinear Approximation Theory will be presented and several open problems will be discussed. The emphasis will be placed on nonlinear n-term approximation from localized frames in various settings such as on the sphere, ball, box and simplex as well as on Lie groups and Riemannian manifolds. Nonlinear approximation from multivariate splines will also be considered. This talk will focus on the characterization of the rates of nonlinear approximation of interest and the related smoothness spaces.

Plenary talk - December 17, 11:00 – 11:55

Differential Groups and the Gamma Function

Michael F. Singer
North Carolina State University, USA
singer@math.ncsu.edu

In 1887, Hoelder proved that the Gamma Function, defined by the difference equation y(x+1) = x y(x), satisfies no nonzero polynomial differential equation with complex coefficients. In the last several years Galois theories have been developed that reprove this result and allow one to characterize when functions satisfying certain linear differential or difference equations also satisfy auxiliary difference or differential equations. These Galois theories allow one to reduce such kinds of questions to questions concerning linear differential or difference groups, that is groups of matrices whose entries are functions satisfying a fixed set of differential or difference equations. I will give an introduction to the theory of these groups and the related Galois theories and survey recent results applying these theories to questions of functional transcendence.

Plenary talk - December 18, 9:30 – 10:25

Models of tumor growth and therapy

Benoit Perthame
Laboratoire J.-L. Lions, Universite Pierre et Marie Curie (Paris 6), France
benoit.perthame@upmc.fr

Models of tumor growth are now commonly used to predict the evolution of cancers, based on images for instance. These models serve to predict the evolution of the disease in medical treatments, to understand the biological effects that permit tumor growth and decide of the optimal therapy. A more recent subject is to explain emergence of resistance to drug and its implication in therapeutic failures.

These models contain several levels of complexity, both in terms of the biological and mechanical effects, and therefore in their mathematical description. The number of scales, from molecules to the organ and entire body, explains partly the complexity of the problem.

In this talk I shall give a general presentation of the field and focus on two aspects. I shall firstly present a multiscale approach to mechanical models of tumor growth and secondly, models of resistance to therapy and treatment optimization.

The part on Hele-Shaw is a collaboration with F.Quiros and J.-L.Vazquez (Universidad Autonoma Madrid), M.Tang (SJTU) and N.Vauchelet (LJLL). The part on adaptation and resistance is a collaboration with O.Diekmann, P.-E.Jabin, S.Mischler, A.Escargueil, J.Clairambault, T.Lorenzi, A.Lorz, G.Barles, S.Mirrahimi, P.E.Souganidis, V.Calvez.

Plenary talk - December 18, 11:00 – 11:55

On Adaptive Multilevel Monte Carlo and Multi-Index Monte Carlo

Raul Tempone
King Abdullah University of Science and Technology, Saudi Arabia
raul.tempone@kaust.edu.sa

We provide a quick glance into recently developed Adaptive Multilevel Monte Carlo (MLMC) Methods for the following widely applied mathematical models: (i) Itô Stochastic Differential Equations, (ii) Stochastic Reaction Networks modeled by Pure Jump Markov Processes and (iii) Partial Differential Equations with random inputs. In this context, the notion of adaptivity includes several aspects such as mesh refinements based on either a priori or a posteriori error estimates, the local choice of different time stepping methods and the selection of the total number of levels and the number of samples at different levels. Our Adaptive MLMC estimator is based on a hierarchy of adaptively refined, non-uniform time discretizations, and, as such, it may be considered a generalization of the uniform discretization MLMC method introduced independently by M. Giles and S. Heinrich. In particular, we show that under some assumptions our adaptive MLMC algorithms are asymptotically accurate and have the correct complexity with an improved control of the complexity constant factor in the asymptotic analysis. Moreover, we show the asymptotic normality of the statistical error in the MLMC estimator, justifying in this way our error estimate that allows prescribing both the required accuracy and confidence level in the final result.

We will show several examples, including some dynamics with singularities and/or non-smooth observables, to illustrate the above results and the corresponding computational savings.

Finally, we will briefly describe the Multi Index Monte Carlo method, presenting new and improved complexity results which are natural generalizations of Giles's MLMC analysis.

Joint work with Nathaniel Collier (Oak Ridge National Laboratory, USA), Abdul Lateef Haji-Ali (King Abdullah University of Science and Technology, Saudi Arabia), Hákon Hoel (University of Oslo, Norway), Alvaro Moraes (King Abdullah University of Science and Technology, Saudi Arabia), Fabio Nobile (Ecole Politechnique Fédérale Lausanne, Switzerland), Erik von Schwerin (Royal Institute of Technology, Sweden), Anders Szepessy (Royal Institute of Technology, Sweden) and Pedro Vilanova (King Abdullah University of Science and Technology, Saudi Arabia).

Plenary talk - December 19, 9:00 – 9:55

AFEM for the Laplace-Beltrami Operator: Convergence Rates

Ricardo H. Nochetto
University of Maryland, USA
rhn@math.umd.edu

Elliptic partial differential equations (PDEs) on surfaces are ubiquitous in science and engineering. We present several geometric flows governed by the Laplace-Beltrami operator. We design a new adaptive finite element method (AFEM) with arbitrary polynomial degree for such an operator on parametric surfaces, which are globally Lipschitz and piecewise in a suitable Besov class: the partitions thus match possible kinks. The idea is to have the surface sufficiently well resolved in W^1_∞ relative to the current resolution of the PDE in H^1. This gives rise to a conditional contraction property of the PDE module and yields optimal cardinality of AFEM. Moreover, we relate the approximation classes to Besov classes.

If the meshes do not match the kinks, or they are simply unknown beforehand, we end up with elliptic PDEs with discontinuous coefficients within elements. In contrast to the usual perturbation theory, we develop a new approach based on distortion of the coefficients in L_q with $q < \infty$. We then use this new distortion theory to formulate a new AFEM for such discontinuity problems, show optimality of AFEM in the sense of distortion versus number of computations, and report insightful numerical results supporting our analysis.

Joint work with A. Bonito (Texas A&M University, USA), M. Cascon (Universidad de Salamanca, Spain), R. DeVore (Texas A&M University, USA), K. Mekchay (Chulalongkorn University, Thailand) and P. Morin (Universidad Nacional del Litoral, Argentina).

Plenary talk - December 19, 11:00 – 11:55

Solving high-dimensional PDEs by tensor product approximation

Reinhold Schneider
TU Berlin , Germany
schneidr@math.tu-berlin.de

Hierarchical Tucker tensor format introduced by Hackbusch et al. including Tensor Trains (TT) (Tyrtyshnikov) have been introduced in 2009. These representations offer stable and robust approximation of high order tensors and multi-variate functions by a low order cost . For many high dimensional problems, including many body quantum mechanics, uncertainty quantification etc., this approach has great potential to circumvent from the *curse of dimensionality.* In case $\mathcal{V} = \bigotimes_{i=1}^{d} \mathcal{V}_i$ the complexity remains proportional to d and polynomial in some multilinear ranks. The approximation properties w.r.t. to these ranks are depending on bilinear approximation rates and corresponding trace class norms. Despite fundamental problems in multilinear approximation, under certain conditions optimal convergence rates

could be shown. The present formats are equivalent to tree tensor networks states and matrix product states (MPS) introduced for the treatment of quantum spin systems. For numerical computations, we cast the PDEs into optimization problems constraint by restricting to set of d tensors of bounded multilinear ranks **r**. The underlying admissible set is no longer convex, but it is an algebraic variety. We consider optimization on Riemannian manifold and corresponding gradient schemes. Numerical examples include electronic Schrödinger equations, uncertainty quantification and molecular dynamics.

Joint work with A. Uschmajew (U Bonn).

Plenary talk - December 20, 9:00 – 9:55

Zeros (of some polynomials) prefer curves

Andrei Martínez-Finkelshtein
University of Almería, Spain
andrei@ual.es

Polynomials satisfying non-Hermitian orthogonality relations appear naturally in approximation theory (as denominators of Padé and Hermite-Padé approximants), random matrix theory, differential equations and special functions, just to mention a few branches of analysis. They are defined by orthogonality relations on the complex plane in which we can freely deform the paths of integration. It is natural to ask where the zeros of these polynomials choose to go. Computation shows that they like to align themselves along certain curves on the plane. What are these curves? In some cases we can answer this question, at least asymptotically. But the answer connects fascinating mathematical objects, such as extremal problems in electrostatics, Riemann surfaces, trajectories of quadratic differentials, algebraic functions; this list is not complete.

This is a brief survey of some ideas related to this problem, starting from the classics, with an emphasis on the breakthrough developments in the 1980-ies, and finishing with some recent results and open problems.

Joint work with E. A. Rakhmanov (University of South Florida, USA).

Plenary talk - December 20, 11:00 – 11:55

Combinatorial Algebraic Geometry

David Cox
Amherst College, USA
dacox@amherst.edu

This talk will survey aspects of the emerging field of combinatorial algebraic geometry. The topics covered will include polytopes involved in quantum computing, formulas for a classical secant variety, likelihood functions, and the combinatorics of a classical moduli space as revealed by tropical geometry. I will also mention some of the computational issues involved.

Workshop A1
Computational Dynamics

Organizers: Hiroshi Kokubu – Rafael de la Llave

A1 - December 11, 14:30 – 14:55

A parametrized Newton-Kantorovich method for rigorously computing (un)stable manifolds: non-resonant and resonant spectra

Christian Reinhardt
VU University Amsterdam, Netherlands
c.p.reinhardt@vu.nl

In this talk we present numerical techniques for the rigorous computation of parametrizations for (un)stable manifolds to hyperbolic fixed points of analytic vector fields via the so called parametrization method. We facilitate our analysis by interpreting the infinite sequence of homological equations derived from the invariance equation fulfilled by the parametrization as an equivalent zero finding problem on the space of geometrically decaying sequences. Using this viewpoint an approach to compute rigorous error bounds on approximate parametrizations is presented. It is based on solving an equivalent parametrized fixed point problem on a ball around a numerical approximation using the contraction principle. In particular our analysis is successful even if internal non-resonance conditions crucial to previous methods fail to hold. We finish the talk by showing example implementations.

Joint work with Jan Bouwe van den Berg (VU University Amsterdam, Netherlands) and Jason Mireles-James (Florida Atlantic University, USA).

A1 - December 11, 15:00 – 15:25

Computing global invariant manifolds of dynamical systems

Pablo Aguirre
Universidad Técnica Federico Santa María, Chile
pablo.aguirre@usm.cl

Global bifurcations of dynamical systems are characterised by the re-arrangement of the higher dimensional stable and unstable manifolds of invariant objects under parameter variation. This may result in drastic changes of the dynamics, including transitions to chaotic regimes, transforming or creating basins of attraction and, ultimately, reorganising the overall structure of the phase space. The aim of this talk is to discuss how the study of global invariant manifolds by analytical and computational methods allows one to obtain deeper insight into the nature of global bifurcations.

A1 - December 11, 15:30 – 15:55

Coexistence of chaos and hyperchaos

Daniel Wilczak
Jagiellonian University, Poland
wilczak@ii.uj.edu.pl

Chaotic attractors can have a very nonuniform internal structure. Even on the plane the well known Newhouse phenomenon guarantees that small sinks of very high periods may be embedded in large chaotic zones. For higher dimensional systems one can expect coexistence of chaotic and hyperchaotic dynamics, i.e. topological horseshoes with more than one positive Lyapunov exponents.

Consider the classical 4D Rossler system

$$\dot{x} = -y - w, \quad \dot{y} = x + ay + z, \quad \dot{z} = dz + cw, \quad \dot{w} = xw + b$$

with the parameter values $a = 0.27857$, $b = 3$, $c = -0.3$, $d = 0.05$. Let

$$\Pi = \{(x, 0, z, w) \in \mathbb{R}^3, \dot{y} = x + z < 0\}$$

be the Poincare section and let $P : \Pi \to \Pi$ be the associated Poincare map.

We prove that

1) there is an explicitly given trapping region $B \subset \Pi$ for P, i.e. $P(B) \subset B$,

2) the maximal invariant set $A = \mathrm{inv}(P, B)$ contains three invariant sets, say H_1, H_2, H_3, on which the dynamics is Σ_2 chaotic, i.e. it is semiconjugated to the Bernoulli shift on two symbols,

3) H_1 is a chaotic set with two positive Lyapunov exponents,

4) H_2 and H_3 are chaotic sets with one positive Lyapunov exponent,

5) there is a countable infinity of heteroclinic connections linking H_1 with H_2, H_2 with H_3 and H_1 with H_3,

6) there is countable infinity of periodic orbits and heteroclinic/homoclinic orbits inside each horseshoe.

Joint work with Roberto Barrio (Universidad de Zaragoza, Spain) and Sergio Serrano (Universidad de Zaragoza, Spain).

A1 - December 11, 16:00 – 16:25

Computer-assisted analysis of Craik's 3D dynamical system

Tomoyuki Miyaji
Meiji University, Japan
tmiyaji@meiji.ac.jp

The following system of equations is studied:

$$\dot{x} = ayz + bz + cy, \dot{y} = dzx + ex + fy, \dot{z} = gxy + hy + kx,$$

where $x(t), y(t)$, and $z(t)$ are real-valued functions, $\dot{x}, \dot{y}$, and $\dot{z}$ are their derivatives with respect to the independent variable t, and the coefficients a to k are real constants. This system arises several contexts in mechanics and fluid mechanics. Especially, Craik has shown that the equations of the form describe a class of exact solutions of the full incompressible Navier-Stokes equations.

Most of solution orbits for the system are unbounded. We can, however, observe characteristic behaviour. A typical solution orbit draws a helical curve, which changes amplitude in a vicinity of the origin. Some solutions change only the amplitude, while some solutions change not only the amplitude but also the axis along which they go to infinity as $t \to \infty$. Craik and Okamoto have found a four-leaf structure and a periodic orbit, which play an important role in controlling the solution orbits. We prove the existence of such a periodic orbit by a method of numerical verification.

A1 - December 11, 17:00 – 17:25

Coexistence of hexagons and rolls

Jean-Philippe Lessard
Université Laval, Canada
jean-philippe.lessard@mat.ulaval.ca

In this talk we introduce a rigorous computational method for finding heteroclinic solutions of a system of two second order differential equations. These solutions correspond to standing waves between rolls and hexagonal patterns of a two-dimensional pattern formation PDE model. After reformulating the problem as a projected boundary value problem (BVP) with boundaries in the stable/unstable manifolds, we compute the local manifolds using the Parameterization Method and solve the BVP using Chebyshev series and the radii polynomial approach. Our results settle a conjecture by Doelman et al. about the coexistence of hexagons and rolls.

Joint work with Jan Bouwe van den Berg (VU Amsterdam), Andréa Deschênes (Laval) and Jason Mireles James (Rutgers).

A1 - December 11, 17:30 – 17:55

Golden mean Siegel disks: renormalization hyperbolicity and applications

Denis Gaidashev
Uppsala University, Sweden
gaidash@math.uu.se

We discuss two different renormalization techniques for golden mean Siegel disks, cylinder renormalization and renormalization for commuting pairs, and describe our recent computer-assisted proof of the renormalization hyperbolicity for almost commuting pairs.

This proof closes the issue of the golden mean universality in Siegel disks, and also leads to existence of golden mean Siegel cylinders in $\mathbb{C}^2$ for a class of two-dimensional perturbations of the golden mean quadratic polynomial.

Joint work with Michael Yampolsky (University of Toronto, Canada).

A1 - December 11, 18:00 – 18:25

Topological Data Analysis on Amorphous Structures

Yasuaki Hiraoka
Kyushu University, Japan
hiraoka@imi.kyushu-u.ac.jp

Description of amorphous structures has been a long standing problem. What is lacking there is an appropriate language to describe geometric structures including short-range order (SRO) and medium-range order (MRO). In this talk, we present computational topological methods based on persistent homology and apply them into atomic arrangements of silica (SiO2) and granular systems. The results elucidate the following new geometric features in amorphous structures: (i) clear identification of amorphous states in silica by using persistence diagrams, (ii) hierarchical relationships between SRO and MRO in silica, (iii) presence of specific packing states (FCC, HCP, BCC, etc) in three dimensional granular packing experiments and transitions among them.

A1 - December 12, 14:35 – 15:25

Developing a computationally efficient algebraic/combinatorial framework for nonlinear dynamics

Konstantin Mischaikow
Rutgers, USA
mischaik@math.rutgers.edu

Typical descriptions of the behavior of nonlinear dynamical systems focus on invariant sets. This leads to an amazingly rich mathematical theory, both in terms of the variety of invariants and the variety of bifurcations between different invariant sets. This richness leads to problems in the context of multiscale applications where one does not have exact models, i.e. the nonlinearities are not known from first principles, or equivalently parameters are unknown or at best poorly known, and in settings in which only crude measurements of the system can be made.

With this in mind I will describe our efforts to develop a computationally efficient, but mathematically rigorous framework for extracting combinatorial/algebraic topological descriptions of the global dynamical structures of multi-parameter nonlinear systems.

A1 - December 12, 15:30 – 15:55

Detecting Morse decompositions of the global attractor of regulatory networks by time series data

Hiroe Oka
Ryukoku University, Japan
oka@rins.ryukoku.ac.jp

Complex network structure frequently appear in biological systems such as gene regulatory networks, circadian rhythm models, signal transduction circuits, etc. As a mathematical formulation of such biological complex network systems, Fiedler, Mochizuki and their collaborators (JDDE 2013) recently defined a class of ODEs associated with a finite digraph called a regulatory network, and proved that its dynamics on the global attractor can in principle be faithfully monitored by information from a (potentially much) fewer number of nodes called the feedback vertex set of the graph.

In this talk, I will use their theory to give a method for detecting a more detailed information on the dynamics of regulatorynetworks, namely the Morse decomposition of its global attractor. The main idea is to take time series data from the feedback vertex set of a regulatory network, and construct a combinatorial multi-valued map, to which we apply the so-called Conley-Morse Database method.

As a test example, we study Mirsky's mathematical model for mammalian circadian rhythm which can be represented as aregulatory network with 21 nodes, and show that numerically generated time series data from its feedback vertex set consisting of 7 nodes correctly detect a Morse decomposition in the global attractor, including 1 stable periodic orbit, 2 unstable periodic orbits, and 1 unstable fixed point.

Joint work with B. Fiedler (Free University of Berlin, Germany), H. Kokubu (Kyoto University, Japan), G. Kurosawa (RIKEN, Japan), and A. Mochizuki (RIKEN, Japan).

A1 - December 12, 16:00 – 16:25

Switching networks database as a platform for parameter search in gene regulatory networks

Tomas Gedeon
Montana State University, USA
gedeon@math.montana.edu

Experimental data on gene regulation and protein interaction is often very qualitative, with the only information available about pairwise interactions is the presence of either up- or down- regulation. Since majority of the parameters for any model in such a situation are not constrained by data, it is important to understand how different choices of parameters affect the dynamics and, therefore, the predictions of such a model. Continuous time Boolean networks, or switching networks, represent an attractive platform for qualitative studies of gene regulation, since the dynamics at fixed parameters is relatively easily to compute. However, it is quite difficult to analytically understand how changes of parameters affect dynamics. On the other hand, the Database for Dynamics is an numerical approach to study global dynamics over a parameter space. The results obtained by this method provably capture the dynamics a predetermined spatial scale. We combine these two approaches to present a method to study switching networks over parameter spaces. We apply our method to experimental data for cell cycle dynamics.

Joint work with Bree Cummins (Montana State University, USA), Bridget Fan (Montana State University, USA), Konstantin Mischaikow (Rutgers University, USA), Arnaud Goullet (Rutgers University, USA), Shaun Harker (Rutgers University, USA).

A1 - December 12, 17:00 – 17:25

An application of the persistence paradigm to the computation of the Conley index

Paweł Pilarczyk
Institute of Science and Technology Austria, Austria
pawel.pilarczyk@ist.ac.at

A new idea for determining the Conley index will be introduced, using an approximate isolating neighborhood and an approximation of a map. The approach is based on the persistence paradigm applied to a sequence of candidates for index pairs, whose relative homology is linked by homomorphisms corresponding to inclusions. A construction will be proposed in which the exit set is defined depending on the percentage of the image that falls outside of the neighborhood. A few sample applications of this method to discrete-time dynamical systems and to flows will be shown. These applications are based on combinatorial sets and outer approximations of maps with respect to a cubical grid in the Euclidean space.

A1 - December 12, 17:30 – 17:55

Analyzing the dynamics of pattern formation in the space of persistence diagrams

Miroslav Kramar
Rutgers University, USA
miroslav@math.rutgers.edu

Persistence diagrams are a relatively new topological tool for describing and quantifying complicated patterns in a simple but meaningful way. We will demonstrate this technique on patterns appearing in Rayleigh-Benard convection. This procedure allows us to transform experimental or numerical data from experiment or simulation into a point cloud in the space of persistence diagrams. There are a variety of metrics that can be imposed on the space of persistence diagrams. By choosing different metrics one can interrogate the pattern locally or globally, which provides deeper insight into the dynamics of the process of pattern formation. Because the quantification is being done in the space of persistence diagrams this technique allows us to compare directly numerical simulations with experimental data.

Joint work with Konstantin Miscahikow (Rutgers, USA), Rachel Levanger (Rutgers, USA), Shaun Harker (Rutgers, USA)., Michael Schatz (Georgia Tech, USA), Jeffrey Tithof (Georgia Tech, USA), Balachandra Suri (Georgia Tech, USA), Mark Paul (Virginia Tech, USA) and Mu Xu (Virginia Tech, USA).

A1 - December 13, 14:35 – 15:25

The Dynamics of Nucleation

Evelyn Sander
George Mason University, United States
esander@gmu.edu

The Cahn-Hilliard equation is one of the fundamental models to describe phase separation dynamics in metal alloys. In this talk, I will focus on applying traditional dynamical tools, such as bifurcation theory and computational topology in order to gain a better understanding of the droplet formation during nucleation for the stochastic Cahn-Hilliard equation. I will consider different types of noise and different types of boundary conditions.

A1 - December 13, 15:30 – 15:55

Rigorous Continuation of Solutions of PDEs

Marcio Gameiro
University of Sao Paulo, Brazil
gameiro@icmc.usp.br

We present a rigorous numerical method to compute solutions of infinite dimensional nonlinear problems. The method combines classical predictor corrector algorithms, analytic estimates and the uniform contraction principle to prove existence of smooth branches of solutions of nonlinear PDEs. The method is applied to compute equilibria and time periodic orbits for PDEs defined on two- and three-dimensional spatial domains.

A1 - December 13, 16:00 – 16:25

Existence of globally attracting solutions of the viscous Burgers equation on the line with periodic boundary conditions and nonautonomous forcing

Jacek Cyranka
Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Poland
jcyranka@gmail.com

We prove the existence of globally attracting solutions of the viscous Burgers equation with periodic boundary conditions on the line for some particular choices of viscosity and non-autonomous forcing

$u_t + u \cdot u_x - \nu u_{xx} = f(t, x)$.

The attracting solution is periodic if the forcing is periodic. The convergence towards attracting solution is exponential. The proof is computer assisted. The method is general and can be applied to other similar partial differential equations including the Navier-Stokes equations.

The technique we use is not restricted to some particular type of equation nor to the dimension one, as we are not using any maximum principles, nor unconstructive functional analysis techniques. We need some kind of 'energy' decay as a global property of our dissipative PDEs and then if the system exhibits an attracting orbit, then we should in principle be able to prove it independent of the dimensionality of of the system. At the present state our technique strongly relies on the existence of good coordinates, the Fourier modes in the considered example. We hope that the further development of the rigorous numerics for dissipative PDEs based on other function bases, e.g. for example the finite elements, should allow to treat also different domains and boundary conditions in near future.

[1] J. Cyranka, P. Zgliczyński, Existence of globally attracting solutions for one-dimensional viscous Burgers equation with nonautonomous forcing - a computer assisted proof, in revision, arXiv:1403.7170.

[2]J. Cyranka, P. Zgliczyński: The effect of fast movement in dissipative PDEs with the forcing term, preliminary preprint, arXiv:1407.1712.

Joint work with Piotr Zgliczyński (Jagiellonian University, Poland).

A1 - December 13, 17:00 – 17:25

Persistent homology of self-maps

Marian Mrozek
Jagiellonian University, Poland
mrozek@ii.uj.edu.pl

When a topological space is known only from sampling, persistence provides a useful tool to study its homological properties. In many applications one can sample not only the space, but also a map acting on the space. The understanding of the topological features of the map is often of interest, in particular in the analysis of time series dynamics but also in the dynamics of a map or differential equation given explicitely when the rigorous study is computationally too expensive and only numerical experiments are available. The aim of the talk is to present an extension of persistent homology to the case of a continuous self-map together with the associated algorithm and numerical examples based on the implementation of the algorithm.

Joint work with Ulrich Bauer (IST, Austria), Herbert Edelsbrunner (IST, Austria) and Grzegorz Jablonski (Jagiellonian University, Poland).

A1 - December 13, 17:30 – 17:55

Discrete and classical vector field dynamics

Tomasz Kaczynski
Université de Sherbrooke, Canada
t.kaczynski@usherbrooke.ca

The Forman's discrete Morse theory is an analogy of the classical Morse theory with, so far, only informal ties. Our goal is to establish a formal tie on the level of induced dynamics. Following the Forman's 1998 paper on "Combinatorial vector fields and dynamical systems", we start with a possibly non-gradient combinatorial vector field. We construct a flow-like upper semi-continuous acyclic-valued mapping whose dynamics is equivalent to the dynamics of the Forman's discrete vector field, in the sense that isolated invariant sets and index pairs are in one-to-one correspondence. The passage to an equivalent continuous single-valued vector field is completed using the graph approximation. This is a joint work with M. Mrozek and Th. Wanner. In my talk, I focus on the construction and properties of the linking multivalued mapping and refer to the Mrozek's talk at the B2 workshop for the extension of the Conley index theory in the combinatorial setting.

Joint work with Marian Mrozek (Jagiellonian University, Krakow, Poland), and Thomas Wanner (George Mason University, Fairfax, USA).

Workshop A2
Computational Harmonic Analysis, Image and Signal Processing

Organizers: Sung Ha Kang – Ursula Molter – Jared Tanner

A2 - December 11, 14:35 – 15:25

Streaming signal reconstruction from generalized measurements

Justin Romberg
Georgia Institute of Technology, USA
jrom@ece.gatech.edu

The central theme of this talk is reconstructing a signal from a stream of generalized samples. This problem has a long history in the signal processing literature. To date, the majority of the results revolve around systems which take samples of and reconstruct the signal using filterbanks with multiple channels, or reconstruct the signal in "batch mode" by collecting a large number of measurements and then perform the inversion of the entire signal by solving a system of linear equations. In the first part of this talk, we will present a method for reconstructing a signal online that lies in between these two approaches. We set the reconstruction up as a linear inverse problem, and then show how to solve the system in an "online" manner.

In the second part of the talk, we show how these ideas can be extended to sparse reconstruction, where we are solving an l1-regularized inverse problem. We present a collection of homotopy-based algorithms that dynamically update the solution of the underlying L1 problem as the system changes.

Joint work with M. Salman Asif.

A2 - December 11, 15:30 – 15:55

On exact recovery of signals from the projection onto polynomial spaces

Shai Dekel
GE Global Research and School of Mathematical Sciences, Tel-Aviv, Israel
shai.dekel@ge.com

In the talk we will review some recent contributions (as well as ours) to the following prototype problem: We are given the projection of a superposition of Diracs onto a finite dimensional polynomial space over a manifold (e.g. trigonometric polynomials, algebraic polynomials, spherical harmonics) and we wish to recover the signal exactly and in particular, the locations of the knots. We will show that under a separation condition on the support of the unknown signal, there exists a unique solution through TV minimization over the space of Borel measures. Time allowing, we will present extensions to recovery of splines, streams of pulses, numerical algorithms, experimental results, stability under noise and more.

Joint work with Tamir Bendory (Technion) and Arie Feuer (Technion).

A2 - December 11, 16:00 – 16:25

Least squares regularized or constrained by L_0: relationship between their optimal solutions and properties

Mila Nikolova
CMLA – CNRS ENS-Cachan, France
nikolova@cmla.ens-cachan.fr

When looking for a sparse solution of an under-determined linear system, two popular options are to find the optimal solution of the least squares regularized by L_0 pseudo-norm using a trade-off parameter β or constrained by L_0 (known also as the K-sparsity constrained problem).

We provide important features of the optimal solutions of both problems. We analyse in depth the relationship between the sets of optimal solutions of these two nonconvex (combinatorial) models. A general quasi-equivalence between these problems is established in the sense explained next. There exists a strictly decreasing sequence of critical values $\{\beta_k\}$ that partitions the positive axis into a certain number of intervals. For every β inside the K-th interval, the regularized problem and the K-constrained problem share exactly the same set of optimal solutions. For $\beta = \beta_k$, the optimal set of the regularized problem is composed of the optimal solutions of the K-constrained problem and the constrained problem corresponding to the next interval (and possibly a few others in-between). All β_k's are obtained from the optimal values of the constrained problems. The strict decay of $\{\beta_k\}$ is a necessary and sufficient condition. We will present all important points concerning this quasi-equivalence.

Small-size exact numerical tests illustrate the theoretical results. By way of conclusion, the K-sparsity problem offers wider possibilities which is not necessarily an advantage.

A2 - December 11, 17:00 – 17:25

Simultaneous high dynamic range image reconstruction and denoising for non-static scenes

Pablo Musé
Universidad de la República, Uruguay
pmuse@fing.edu.uy

The human eye has the ability to capture scenes of very high dynamic range, retaining details in both dark and bright regions. This is not the case for current standard digital cameras. The limited capacity of the sensor cells makes it impossible to record the irradiance from very bright regions for long exposures (saturated pixels) while very few photons will be captured in the dark regions for short exposures (noisy pixels).

Since the seminal work of Mann and Picard in 1994, the standard way to build high dynamic range (HDR) images from regular cameras has been to combine a reduced number of photographs captured with different exposure times. The algorithms proposed in the literature differ in the strategy used to combine these frames.

Under the hyphotesis of perfectly aligned images (fixed scene and static camera), a study of the different noise sources in the image acquisition process allows us to model the image fusion as a statistical estimation problem. We derive theoretical bounds for the performance of the HDR estimation problem and show that, even with a small number of photographs, the maximum likelihood estimator performs extremely close to these bounds.

In practice, scenes are dynamic and images are usually acquired with a hand-held camera. We propose a new HDR image generation approach that simultaneously copes with these problems and exploits image redundancy to produce a denoised result. A reference image is chosen and a patch-based approach is used to find similar pixels that are then combined for the irradiance estimation. This patch-based approach permits to obtain a denoised result and is robust to image misalignments and object motion. Results show significant improvements in terms of noise reduction over previous HDR image generation techniques, while being robust to motion and changes between the exposures.

Joint work with Cecilia Aguerrebere (Duke University, USA), Julie Delon (Télécom ParisTech, France) and Yann Gousseau (Télécom ParisTech, France).

A2 - December 11, 17:30 – 17:55

Denoising an Image by Denoising its Curvature

Stacey Levine
Duquesne University, USA
sel@mathcs.duq.edu

In this work we argue that when an image is corrupted by additive noise, its curvature image is less affected by it. In particular, we demonstrate that for sufficient noise levels, the PSNR of the curvature image is larger than that of the original image. This leads to the speculation that given a denoising method, we may obtain better results by applying it to the curvature image and then reconstructing from it a clean image, rather than denoising the original image directly. Numerical experiments confirm this for several PDE-based and patch-based denoising algorithms.

Joint work with Marcelo Bertalmío (Universitat Pompeu Fabra).

A2 - December 11, 18:00 – 18:25

Recent algorithmic and theoretical advances on graph matching.

Marcelo Fiori
Universidad de la República, Uruguay
mfiori@fing.edu.uy

Problems related with graph matching are very important both from a theoretical and practical point of view, with several applications from image and video analysis to biological and biomedical problems. In this talk we will cover both aspects of this challenging problem.

First, a graph matching algorithm will be presented, combining sparsity ideas with relaxations of the graph matching problem, yielding a robust method, particularly well suited for multimodal data. Then, new probabilistic results about convex and non-convex relaxations of the graph matching problem will be discussed, also suggesting some practical considerations. Finally, new spectral results will be presented, proving the equivalence for the graph matching problem and a relaxed version for certain graphs, also shedding light to the relationship between spectral properties and the automorpshism group of a graph.

Joint work with Guillermo Sapiro (Duke University, USA), Pablo Sprechmann (New York University, USA), Joshua Vogelstein (Duke University, USA), Pablo Musé (Universidad de la República, Uruguay), Vince Lyzinski (Johns Hopkins University, USA), Donniell Fishkind (Johns Hopkins University, USA) and Carey E. Priebe (Johns Hopkins University, USA)..

A2 - December 11, 18:30 – 18:55

Matrix recovery from coarse observations

Mark Davenport
Georgia Institute of Technology, USA
mdav@gatech.edu

In this talk I will describe a theory and techniques for solving matrix completion and similar problems when given extremely coarse (e.g., 1-bit) observations. As an example, instead of observing a subset of the real-valued entries of a matrix M, we might obtain a small number of binary (1-bit) measurements generated according to a probability distribution determined by the real-valued entries of M. The central question I will discuss is whether or not it is possible to obtain an accurate estimates of M from data of this form. In general this would seem impossible, but I will show that the maximum likelihood estimate under a suitable constraint returns an accurate estimate of M under certain natural conditions. I will conclude by discussing several extensions and applications of these techniques to similar problems.

Joint work with Yaniv Plan (University of British Columbia), Ewout van den Berg (IBM Watson) and Mary Wootters (Carnegie Mellon University).

A2 - December 12, 14:30 – 14:55

Fundamentals of dynamical sampling

Carlos Cabrelli
Dept. of Mathematics, FCEyN, UBA and IMAS-UBA-CONICET, Argentina
cabrelli@dm.uba.ar

Dynamical sampling refers to the process that results from sampling an evolving signal f at various times. The fundamental question of this spatial-temporal sampling is: when do coarse samplings taken at varying times contain the same information as a finer sampling taken at the earliest time? In other words, under what conditions on an evolving system, can time samples be traded for spatial samples?

Because dynamical sampling uses samples from varying time levels for a single reconstruction, it departs from classical sampling theory in which a signal f does not evolve in time and is to be reconstructed from its samples at a single time $t = 0$.

In this talk we study this problem in finite dimensional spaces, and for a large class of self adjoint operators in infinite dimensional spaces.

Joint work with Akram Aldroubi (Vanderbilt University, US), Ursula Molter (University of Buenos Aires, Argentina), Sui Tang (Vanderbilt University, US)..

A2 - December 12, 15:00 – 15:25

On spectrogram local maxima

Patrick Flandrin
CNRS & Ecole normale supérieure de Lyon, France
flandrin@ens-lyon.fr

In close connection with time-frequency uncertainty relations, spectrograms have some built-in redundancy which constrains the landscape of their surface, thus calling for simplified descriptions based on a reduced number of salient features. This is investigated in some detail for the distribution of local maxima in the generic case of white Gaussian noise. A simple model, based on a randomized hexagonal lattice structure, is proposed for such a distribution considered as a spatial point process in the time-frequency plane. The rationale of the model is discussed, its relevance is tested with respect to the cumulative distribution function of nearest-neighbour distance between local maxima, and the deviation from the complete spatial randomness of an equivalent Poisson model is quantified. Attaching a Voronoi tessellation to local maxima ends up with a constrained distribution of cells that reflects uncertainty and paves the way for the modeling of spectrogram enhancements such as those offered by reassignment or synchrosqueezing.

A2 - December 12, 15:30 – 15:55

High dimensional learning rather than computing in quantum chemistry

Matthew Hirn
Ecole normale superieure, France
matthew.hirn@ens.fr

Physical functionals are usually computed as solutions of variational problems or from solutions of partial differential equations, which may require huge computations for complex systems. Quantum chemistry calculations of molecular ground state energies is such an example. Machine learning algorithms do not simulate the physical system but estimate solutions by interpolating values provided by a training set of known examples. However, precise interpolations may require a number of examples that is exponential in the system dimension, and are thus intractable. This curse of dimensionality may be avoided by computing interpolations in smaller approximation spaces, which take advantage of physical invariants. We introduce deep multiscale learning architectures in a similar vein to deep neural networks, which compute such invariant approximations via iterated wavelet transforms. Theoretical results relating these architectures to the Coulomb potential from classical physics will motivate numerical applications for molecular energies in quantum chemistry, in relation with Density Functional Theory.

Joint work with Stephane Mallat (Ecole normale superieure, France) and Nicolas Poilvert (Pennsylvania State University, USA).

A2 - December 12, 16:00 – 16:25

Estimation of bandlimited stochastic operators using SIC-POVMs

Götz Pfander
Jacobs University Bremen / Katholische Universität Eichstätt, Germany
g.pfander@jacobs-university.de

We describe a novel estimator for bandlimited stochastic operators. The operators considered are frequently used to model randomly varying parts of radar targets and communications channels. The presented estimator is based on insights on stochastic modulation spaces, in operator sampling theory, and on finite dimensional Gabor frames. In case of wide sense stationary with uniform scattering (WSSUS) targets, we exhibit a connection to symmetrically information complete positive operator valued measures (SIC-POVMs) as considered in quantum information theory.

Joint work with Pavel Zheltov (Jacobs University Bremen).

A2 - December 12, 17:00 – 17:25

Consistency of probability measure quantization by means of power repulsion-attraction potentials

Massimo Fornasier
Technische Universität München, Germany
massimo.fornasier@ma.tum.de

In this talk we are concerned with the study of the consistency of a variational method for probability measure quantization, deterministically realized by means of a minimizing principle, balancing power repulsion and attraction potentials. The proof of consistency is based on the construction of a target energy functional whose unique minimizer is actually the given probability measure to be quantized. Then we show that the discrete functionals, defining the discrete quantizers as their minimizers, actually Gamma-converge to the target energy with respect to the narrow topology on the space of probability measures. A key ingredient is the reformulation of the target functional by means of a Fourier representation, which extends the characterization of conditionally positive semi-definite functions from points in generic position to probability measures.

Joint work with Jan-Christian Hütter (MIT, USA).

A2 - December 12, 18:05 – 18:55

Weighted sparsity for function approximation and interpolation

Rachel Ward
University of Texas at Austin, USA
rward@math.utexas.edu

Functions of interest are often smooth and sparse in some sense, and both priors should be taken into account when interpolating sampled data. Linear interpolation methods are effective under strong regularity assumptions, but do not incorporate nonlinear sparsity structure. At the same time, nonlinear methods such as ℓ_1 minimization can reconstruct sparse functions from very few samples, but do not encourage smoothness without adding weights. We argue that weighted ℓ_1 minimization effectively merges the two approaches, promoting both sparsity and smoothness in reconstruction. Along the way, we introduce a notion of weighted sparsity and extend concepts from compressive sensing such as the restricted isometry property and null space property to accommodate weighted sparse expansions. We expect these developments to be of independent interest in the study of structured sparse approximations and continuous-time compressive sensing problems.

Joint work with Holger Rauhut (Aachen University).

A2 - December 13, 14:30 – 14:55

Color Stabilization Along Time and Across Shots of the Same Scene, for One or Several Cameras of Unknown Specifications

Marcelo Bertalmío
Universitat Pompeu Fabra, Spain
marcelo.bertalmio@upf.edu

We propose a method for color stabilization of shots of the same scene, taken under the same illumination, where one image is chosen as reference and one or several other images are modified so that their colors match those of the reference. We make use of two crucial but often overlooked observations: firstly, that the core of the color correction chain in a digital camera is simply a multiplication by a 3x3 matrix; secondly, that to color-match a source image to a reference image we don't need to compute their two color correction matrices, it's enough to compute the operation that transforms one matrix into the other. This operation is a 3x3 matrix as well, which we call H. Once we have H, we just multiply by it each pixel value of the source and obtain an image which matches in color the reference. To compute H we only require a set of pixel correspondences, we don't need any information about the cameras used, neither models nor specifications or parameter values. We propose an implementation of our framework which is very simple and fast, and show how it can be successfully employed in a number of situations, comparing favourably with the state of the art. There is a wide range of applications of our technique, both for amateur and professional photography and video: color matching for multi-camera TV broadcasts, color matching for 3D cinema, color stabilization for amateur video, etc.

Joint work with Javier Vazquez-Corral (Universitat Pompeu Fabra, Spain).

A2 - December 13, 15:00 – 15:25

Multi-level structured sparse models

Pablo Sprechmann
Courant Institute, New York University, United States
pablo.sprechmann@nyu.edu

Parsimony, including sparsity and low rank, has been shown to successfully model data in numerous machine learning and signal processing tasks. Sparse models assume minimal prior knowledge about the data, asserting that the signal has many coefficients close or equal to zero when represented in a given domain. From a data modeling point of view, sparsity can be seen as a form of regularization, that is, as a device to restrict or control the set of coefficient values which are allowed in the model to produce an estimate of the data. While this model has been proven to be effective in many settings, it is found insufficient in difficult inverse problems such as source separation. In these situations, one could greatly benefit from learning further structure present in the data. We propose a new type of structure sparse models that aim at learning a multi-level sparse representation of the data. The proposed representation is organized hierarchically and aims at learning high-level structure, such as dependencies (or correlations) in the activations and short-term temporal dynamics. We evaluate the proposed model on a monaural audio separation task and discuss connections with deep learning.

Joint work with Joan Bruna (New York University, USA) and Yann Lecun (New York University, USA).

A2 - December 13, 15:35 – 16:25

On the stability of least-squares approximations. Application in acoustics.

Albert Cohen
Université Paris VI, France
cohen@ann.jussieu.fr

A2 - December 13, 17:00 – 17:25

Texture aware video inpainting of complex scenes

Andrés Almansa
CNRS & Telecom ParisTech, France
andres.almansa@telecom-paristech.fr

We present an automatic video inpainting algorithm which relies on the multi-scale optimisation of a patch-based functional enforcing self-similarity. Unlike previous approaches, the best patch candidates are selected using persistent multi-scale texture attributes. We show that this rationale prevents the usual wash-out of textured and cluttered parts of videos. The resulting approach is able to successfully and automatically inpaint complex situations, including high resolution sequences with dynamic textures and multiple moving objects.

Joint work with Alasdair Newson (Duke University), Matthieu Fradet (Technicolor), Yann Gousseau (Telecom ParisTech) and Patrick Perez (Technicolor).

A2 - December 13, 17:30 – 17:55

Self similarity and spectral correlation adaptive algorithm for image interpolation

Antoni Buades
Universitat Illes Balears, Spain
toni.buades@uib.es

Most common cameras use a CCD sensor device measuring a single color per pixel. The other two color values of each pixel must be interpolated from the neighboring pixels in the so-called demosaicking process. Recent devices include also grey level sensors in order to increase the signal to noise ratio. These recent devices get closer to the satellite acquisition systems, where the color and luminance information are acquired separately, and with different resolutions; a high-resolution luminance image and a low-resolution multispectral one.

In this talk, I will review classical solutions to these two different interpolation problems and present new algorithms.

Joint work with J. Duran (Universitat Illes Balears, Spain).

A2 - December 13, 18:00 – 18:25

A one stage Sigma-Delta decoder for compressed sensing measurements

Rongrong Wang
University of British Columbia, Canada
rongwang@math.ubc.ca

We analyze how efficiently Sigma-Delta quantization works for quantizing compressed (sub-Gaussian) measurements of sparse and compressible signals. To this end, we propose a one-stage reconstruction algorithm based on convex optimization that yields consistent reconstruction. The algorithm works in the cases of fine and coarse quantization including one-bit quantization, with a reconstruction error decaying inverse polynomially in the quantization order. We show that this decay rate is nearly optimal

among all possible reconstruction algorithms by a geometric argument about quantization cells. When we optimize over all quantization orders, the algorithm can achieve root exponential error decay with respect to the "oversampling factor". Finally, we show that by further compressing the quantized data via a Johnson-Lindenstrauss embedding, exponential decay (as a function of the total bit budget) is achieved.

Joint work with This is joint work with Rayan Saab (UCSD) and Ozgur Yilmaz (UBC).

A2 - December 13, 18:30 – 18:55

Orthonormal Bases Generated by Cuntz Algebras

Myung-Sin Song
Southern Illinois University Edwardsville, USA
msong@siue.edu

We show how some orthonormal bases can be generated by representations of the Cuntz algebra such as Fourier bases on fractal measures, generalized Walsh bases on the unit interval and piecewise exponential bases on the middle third Cantor set.

Joint work with Dorin Dutkay (University of Central Florida, USA) and Gabriel Picioroaga (University of South Dakota, USA).

A2 - Poster

Local convergence of an algorithm for subspace identification with missing data

Laura Balzano
University of Michigan, USA
girasole@umich.edu

Low-dimensional linear subspace approximations to high-dimensional data find application in a great variety of applications where missing data are the norm, not only because of errors and failures in data collection, but because it may be impossible to collect and process all the desired measurements.

In this poster, I will describe recent results on estimating subspace projections from incomplete data. I will discuss the convergence guarantees and performance of the algorithm GROUSE (Grassmannian Rank-One Update Subspace Estimation), a subspace tracking algorithm that performs gradient descent on the Grassmannian. I will also discuss the relationship of GROUSE with an incremental SVD algorithm, and show results of GROUSE applied to problems in computer vision.

Joint work with Stephen J. Wright.

A2 - Poster

Finitely generated shift invariant spaces with extra invariance nearest to observed data

Carolina Mosquera
Universidad de Buenos Aires, CONICET, Argentina
mosquera@dm.uba.ar

Let $m, \ell \in \mathbb{N}$, M be a closed subgroup of $\mathbb{R}^d$ containing $\mathbb{Z}^d$ and $F = \{f_1, \ldots, f_m\} \subset L^2(\mathbb{R}^d)$. We study the problem of finding the shift invariant space V of length less or equal to ℓ which is also $M-$invariant such that V is "closest" to the functions F in the sense that

$$V = argmin_{V' \in V_M^\ell} \sum_{j=1}^{m} \|f_j - P_{V'} f_j\|^2,$$

where V_M^ℓ is the set of all shift invariant spaces V' of length less or equal to ℓ which are also $M-$invariant, and $P_{V'}$ is the orthogonal projection on V'.

Also we consider this problem for a particular set of translation invariant spaces.

Joint work with Carlos Cabrelli (Universidad de Buenos Aires, CONICET, Argentina).

A2 - Poster

Relax, no need to round: Integrality of clustering formulations.

Soledad Villar
University of Texas at Austin, United States
mvillar@math.utexas.edu

When dealing with combinatoric NP-hard problems one can either find an efficient algorithm that works in every case but its solution is not optimal (this is the philosophy behind approximation algorithms) or efficiently compute a probably optimal solution of the original problem under more restrictive hypothesis (which is the spirit of compressed sensing recovery guarantees).

In this work we take the second approach applied to point cloud clustering problems; showing recovery conditions of an integral solution through their convex relaxations. We focus on two of the most common optimization problems for unsupervised clustering: k-means and k-medians clustering.

Joint work with P. Awasthi (Princeton University), A. S. Bandeira (Princeton University), M. Charikar (Princeton University), R. Krishnaswamy (Princeton University) and R. Ward (University of Texas at Austin).

Workshop A3
Computational Number Theory

Organizers: Kristin Lauter – Christophe Ritzenthaler – Peter Stevenhagen

---

A3 - December 11, 14:30 – 15:10

Class invariants for abelian surfaces

Andreas Enge
INRIA Bordeaux-Sud-Ouest, France
andreas.enge@inria.fr

Abelian surfaces, or equivalently Jacobians of genus 2 hyperelliptic curves, are the "next complex" case of abelian varieties after elliptic curves, and a serious contender for fast implementations of cryptosystems based on the discrete logarithm problem. Complex multiplication provides an interesting way of obtaining such surfaces with a known number of points over a finite field; it requires the costly computation of a large polynomial defining a certain class field.

I will report on joint work with Emmanuel Thomé on an asymptotically quasi-linear algorithm to compute such class polynomials with floating point approximations, as well as on its freely available implementation. Then I will present joint work with Marco Streng on a systematic approach for obtaining class invariants, that is, elements of the same class field with smaller minimal polynomials, and show how to solve the problem of obtaining all their algebraic conjugates in an easy way.

Joint work with Emmanuel Thomé (INRIA Nancy-Grand Est, France) and Marco Streng (Universiteit Leiden, Netherlands).

A3 - December 11, 15:10 – 15:50

Aspects of Belyi maps

Jeroen Sijsling
Dartmouth College, United States of America
sijsling@gmail.com

A Belyi map is a finite morphism to the complex projective line that is branched above at most three points. Surprisingly, the algebraic curves that admit a Belyi map are exactly those that are defined over the algebraic closure of the rationals. Due to the simple combinatorial description of covers as finite sets with an action of the fundamental group, the theory of Belyi maps therefore gives a way to study the absolute Galois group of the rationals, one of Grothendieck's dreams.

This talk will explain the links between Belyi maps and other areas of study, such as Shimura curves, inverse Galois theory, and number fields with small ramification. Hopefully this will make show how Belyi maps, like (and linked with) modular forms, can be a useful tool for any computationally inclined number theorist. Finally, the currently available techniques to compute Belyi maps are described, including a recent one due to Klug, Musty, Schiavone and Voight.

Joint work with John Voight.

A3 - December 11, 15:50 – 16:30

Distribution of traces of genus 3 curves

Christophe Ritzenthaler
University Rennes 1, France
ritzenthalerchristophe@gmail.com

We present numerical experiments to visualize the asymmetry in the distribution of traces of Frobenius of genus 3 curves over finite fields. These observations are linked to the so-called Serre obstruction for non hyperelliptic curves. We also give a heuristic explaination for the phenomena we observe.

Joint work with Reynald Lercier (university Rennes 1), Florent Rovetta (university Aix-Marseille), Jeroen Sijsling (Dartmouth college) and Ben Smith (Polytechnique Paris).

A3 - December 11, 17:05 – 17:55

Polynomial time computation of Galois representations attached to modular forms.

Bas Edixhoven
Universiteit Leiden, Netherlands
edix@math.leidenuniv.nl

Modular forms give rise to number fields with non-solvable Galois groups, acting faithfully on finite subgroups of jacobian varieties of curves. In joint work with Couveignes, de Jong and Merkl, generalised by Bruin, it was shown that these number fields can be computed in polynomial time. The major difficulty is that such computations must be done in time polynomial in the dimension of the jacobian varieties that arise. This difficulty was solved by approximate computations and bounds that allow us to get exact solutions from approximate ones. The bounds will be discussed briefly. We will focus on Couveignes's algorithms for the approximate computations. Finally, real computations by Bosman, Mascot, Zeng, Derickx, van Hoeij and Peng will be presented.

Joint work with Jean-Marc Couveignes (Université Bordeaux 1), Robin de Jong (Universiteit Leiden), Franz Merkl (Universität München) and Peter Bruin (Universiteit Leiden).

A3 - December 11, 18:00 – 18:40

Computations on a conjecture of BSD type postulated by B. Mazur and J. Tate

Francisco Portillo
UACM, Mexico
francisco.portillo@uacm.edu.mx

In a series of articles, Mazur and Tate postulated a p-adic analogue of the Birch and Swinnerton-Dyer conjecture for finite layers. This conjecture have similar invariants as the classical BSD conjecture, but it has a multiplicative form. Invariants like the Tamawaga numbers, the order of the torsion, the order of the Tate-Shafarevich group, appear as exponents of the arithmetic side of the conjectured equation. In the analytic side, we have Modular Symbols that play the role of the $\mathcal{L}$ function, and the equation holds over a finite abelian multiplicative group. We will present computational evidence in favor of the mentioned conjecture, and we will explain how it was computed.

A3 - December 12, 14:30 – 15:10

All del Pezzo surfaces of degree two over finite fields are unirational

Cecilia Salgado
UFRJ, Brazil
salgado@im.ufrj.br

A consequence of the Segre-Manin theorem is that a del Pezzo surface of degree two is unirational over its base field as long as it possesses a general rational point defined over the field in question. In this work, joint with D. Testa and A. Várilly-Alvarado, we show that all del Pezzo surfaces of degree two over a finite fields are unirational with at most three possible exceptions. Recently, Festi and van Luijk showed that these three last surfaces are also unirational. I will discuss the arguments involved in our proof.

Joint work with Damiano Testa (Warwick, UK) and Anthony Várilly-Alvarado (Rice, USA).

A3 - December 12, 15:10 – 15:50

Concurrent lines on del Pezzo surfaces of degree one

Ronald van Luijk
Universiteit Leiden, Netherlands
rmluijk@gmail.com

Let k be a field and $\overline{k}$ an algebraic closure. A del Pezzo surface over k is a surface over k that is isomorphic over $\overline{k}$ to either $\mathbb{P}^1 \times \mathbb{P}^1$ (degree 8), or $\mathbb{P}^2$ blown up at $r \leq 8$ points in general position (degree $9 - r$). Famous examples (with $r = 6$ and degree 3) are smooth cubic surfaces in $\mathbb{P}^3$, which over $\overline{k}$ contain 27 lines; at most three of these can be concurrent, that is, go through the same point. Analogously, we get 240 lines for $r = 8$ and degree 1. Based on the graph on these lines, with edges between those that intersect, we get an upper bound of 16 for the number of concurrent lines. We show that this upper bound is only attained in characteristic 2, which makes the case $r = 8$ different from all other cases. In most characteristics, including characteristic 0, the upper bound is 10.

Joint work with Rosa Winter (Universiteit Leiden, Netherlands).

A3 - December 12, 15:50 – 16:30

Computing twists of Shioda modular surfaces of level 4 related to visibility of Sha

Nils Bruin
Simon Fraser University, Canada
nbruin@sfu.ca

One of the most mysterious objects associated to an elliptic curve E is its Tate-Shafarevich group $Sha(E)$. Its elements can be represented by classes in the Galois-cohomology group $H^1(Q, E[n])$, for various n.

If two distinct elliptic curves E and E' have isomorphic n-torsion, then a single class ξ in $H^1(Q, E[n])$ can represent a trivial element in $Sha(E)$ and a non-trivial one in $Sha(E')$. In the terminology of Mazur, the element of $Sha(E')$ is made *visible* by E. Mazur showed that for $n = 3$, all elements of Sha can be made visible. In general, the question translates into whether a rational point lies on a certain twist of the Shioda modular surface, obtained by taking the universal elliptic curve over the modular curve $X(n)$ of full level n.

The case $n = 4$ is particularly interesting. The curve $X(4)$ is rational, but the relevant surface over it is not. It is a K3 surface. Further complications in determining the correct surface arise from the fact that 4 is even. We will discuss how to compute a model of the relevant surface given ξ and give some examples of the various obstructions to rational points that can arise on these surfaces.

Joint work with Tom Fisher (Cambridge, United Kingdom).

A3 - December 12, 17:00 – 17:40

Prime densities for GL_1 and GL_2

Peter Stevenhagen
Universiteit Leiden, Netherlands
psh@math.leidenuniv.nl

If we fix a rational number x, Artin's basic question "for how many primes p does x mod p generate the multiplicative group of non-zero integers modulo p?" leads to Artin's conjecture on primitive roots, and the associated prime density depends in a somewhat non-trivial way on x. A conceptual way to compute such densities is given by the character sum method that I developed with Moree and Lenstra, and that exploits Galois representations coming from the multiplicative group.

Artin-type questions also exist in an elliptic setting, as do the associated Galois representations. I will explain how our character sum method extends to this case.

Joint work with Pieter Moree (Max Planck Institut Bonn) and Hendrik Lenstra (Universiteit Leiden).

A3 - December 12, 17:40 – 18:20

Computing tables of elliptic curves

Ariel Pacetti
UBA, Argentina
apacetti@dm.uba.ar

In modern computational number theory, the existence of tables of elliptic curves plays a central role. They allow to test many open conjectures and give some hint on the behavior of rank and many other quantities of elliptic curves as the conductor grows.

The main contribution on elliptic curves' tables are Cremona's tables, which are based con computing modular symbols and the action of the Hecke operators on them. The problem is that elliptic curves correspond to rational eigenvalues, while most eigenvalues are not rational, so the cost of computing the whole Hecke operators is too big for the few curves obtained. There is a variant due to Cremona-Lingham, which consists on computing j-invariants over number fields, and have some computational cost. In this talk we will present a different approach which consists on using information of the residual 2-adic representation. This allows to speed up computations assuming some conjectures on minimal models up to isogenies. This is a work in progress, and the ideas presented should be generalizable to arbitrary number fields.

A3 - December 13, 14:30 – 15:10

On the number of points of Jacobians over finite fields: from asymptotic theory to applications

Alexey Zykin
Université de la Polynésie française, France
alzykin@gmail.com

Asymptotic theory of global fields was developed by Tsfasman and Vladuts in 1990's in connection with the problem of bounding the number of points on varieties over finite fields and its applications to the coding theory. In my talk I will explain how explicit versions of Tsfasman and Vladuts results (namely, that of the generalized Brauer-Siegel theorem) can be used for getting very tight bounds for the number of points on jacobians of curves over finite fields. If time permits, I will discuss some progress in finer asymptotic questions related to the asymptotically bad situation in the case of cyclotomic fields and modular curves. This is a joint work with Philippe Lebacque.

Joint work with Philippe Lebacque (Université de Franche-Comté, France).

A3 - December 13, 15:10 – 15:50

Arithmetic geometry and key exchange : compact Diffie–Hellman with efficient endomorphisms

Benjamin Smith
INRIA and École polytechnique, France
smith@lix.polytechnique.fr

Diffie–Hellman key exchange is one of the fundamental primitives in public-key cryptography. If G is an abelian group (written additively), then the Diffie–Hellman protocol in G is composed of four computations in the form $P \longmapsto [m]P = P + \cdots + P$ (m times) for various points P and integers m; optimising this scalar multiplication operation is crucial.

In practice, the most efficient contemporary Diffie–Hellman implementations are based on elliptic curves, or Jacobians of genus 2 curves. But in these groups, computing $-P$ is extremely efficient, so we can use the fact that $[m](\pm P) = \pm([m]P)$ to simplify and speed up the protocol, identifying P with $-P$ (formally, working in the quotient set $G/\langle\pm 1\rangle$). These "compact" systems offer significant savings in both space, which translates into slightly shorter keys, and in computing time, through simpler pseudo-group law formulae. In the elliptic curve context, this amounts to using only x-coordinates of points and Montgomery's pseudo-group law. Bernstein's Curve25519 software, which has become a de facto reference implementation of Diffie–Hellman at the 128-bit security level, is a practical example of these techniques in practice. The genus 2 analogue is Kummer surface arithmetic, where we can use particularly efficient formulae developed by the Chudnovskys, and popularized in cryptography by Gaudry.

Recent years have seen renewed interest in the Gallant–Lambert–Vanstone (GLV) technique for computing $[m]P$ in G. Here, we suppose our elliptic curve (or our genus 2 Jacobian) has an efficiently computable non-integer endomorphism ϕ, which when applied to elements of G acts like $[\lambda]$ (for some large eigenvalue λ). Suppose we want to compute $[m]P$: first we use the Euclidean algorithm to compute much smaller integers a and b such that $a + b\lambda \equiv m \pmod{\#G}$, and then we compute $[m]P = [a]P + [b]\phi(P)$. The running time of the multiexponentiation depends on the size of a and b, while traditional scalar multiplication depends on the size of m. In practice, a and b have half the bitlength of m, which means that GLV and its variants can offer us a significant speedup.

In this talk, we will discuss the adaptation of GLV techniques to x-coordinate-only and Kummer surface systems. On the practical side, we will present recent experimental results for a new elliptic-curve based implementation. On the more theoretical side, we will present some new formulae for Kummer surface systems.

A3 - December 13, 15:50 – 16:30

Paramodular forms: central values of twisted spin L-functions

Gonzalo Tornaria
Universidad de la República, Uruguay
tornaria@cmat.edu.uy

In the 1980s Böcherer formulated a conjecture relating the central values of the imaginary quadratic twists of the spin L-function attached to a Siegel modular form F to the Fourier coeficients of F.

In this talk I will present some recent generalizations of this conjecture to the case of paramodular forms, and the computations providing numerical evidence for the new conjectures.

Joint work with Nathan Ryan (Universidad de la República / Bucknell University).

A3 - December 13, 17:00 – 17:40

Hypergeometric Motives

Fernando Rodriguez Villegas
Abdus Salam International Center for Theoretical Physics, Italy
villegas@ictp.it

The families of motives of the title arise from classical one-variable hypergeometric functions. This talk will focus on the calculation of their corresponding L-functions. These represent a fairly wide class of L-functions that are numerically accessible going well beyond standard cases.

Joint work with Frits Beukers (University of Utrecht), Henri Cohen (Universite de Bordeaux), Anton Mellit (ICTP), David Roberts (University of Minnesota, Morris), Masha Vlasenko (University College Dublin) and Mark Watkins (MAGMA group, University of Sydney).

A3 - December 13, 17:40 – 18:20

Torsion structures of elliptic curves over number fields

Filip Najman
University of Zagreb, Croatia
fnajman@math.hr

We say that a torsion structure on an elliptic curve over a number field K is either a K-rational torsion subgroup or a $\mathrm{Gal}(\overline{K}/K)$-invariant cyclic subgroup (which is also the kernel of a cyclic isogeny defined over K) of E. Recent years have seen a great deal of progress in many directions, by work of many people, in understanding torsion structures of elliptic curves over number fields.

I will talk about some of these recent results: which torsion structures are possible over number fields of fixed degree or over certain fixed number fields, about elliptic curves with special“ torsion structures and how they were constructed, which properties can prescribing the torsion structure imbue on an elliptic curve with that torsion structure, and about the number of twists with large torsion that an elliptic curve can have.

Joint work with Peter Bruin (Universiteit Leiden, Netherlands).

A3 - Poster

Congruences between modular forms modulo prime powers

Maximiliano Javier Camporino
Universidad de Buenos Aires, Argentina
mcamporino@dm.uba.ar

Consider the following problem: given a modular form f and a prime power p^n, is there a modular form g, different from f, such that f and g are congruent modulo p^n?. A way to solve this problem is to consider the Galois representation attached to f, take its modulo p^n reduction and study the obstructions appearing when trying to lift it back to a ring of characteristic zero. In this way, we obtain a local-to-global lifting result for abstract representations, which, when combined with the appropiate modularity lifting theorem, translates into an affirmative response for the proposed problem. Moreover, the method provides a way to control the local behavior of the representations (and hence the modular forms) constructed, giving applications to level lowering and level raising problems.

We can show in a concrete example how the method works, getting a case of modulo p^n level raising. We find a congruence modulo 25 between an elliptic curve of conductor 17 and a modular form of level $17 \cdot 113$. The obtainment of this example involves computing groups of Galois cohomology of the representation attached to the elliptic curve and understanding the local behavior of the elements lying inside them. For this, we compute abelian extensions of a high degree Galois extension of $\mathbb{Q}$, which turns out to be a computationally challenging problem.

Joint work with Ariel Pacetti (Universidad de Buenos Aires, Argentina).

A3 - Poster

Heegner Points on Cartan non-split Curves

Daniel Kohen
IMAS-CONICET , Argentina
kohendaniel@gmail.com

The goal is to construct Heegner Points on elliptic curves over $\mathbb{Q}$ in cases where the classical Heegner hypothesis does not hold. Concretely, let $E/\mathbb{Q}$ be an elliptic curve of conductor N , p an odd prime such that p^2 divides N exactly, and K an imaginary quadratic field in which p is inert and the other primes dividing the conductor are split. In this case there aren't any Heegner points in the modular curve $X_0(N)$, but since $sign(E/K) = -1$ we still expect to somehow construct "Heegner points". The idea is to consider other modular curves, the so called Cartan non-split curves, whose Jacobian is isogenous to the new part of $J_0(p^2)$. In order to compute the Abel-Jacobi map we need to compute the Fourier expansions of newforms associated to Cartan non-split groups. These Fourier expansions have coefficients in $\mathbb{Q}(\xi_p)$ and, under a situable normalization, the coefficients satisify nice properties when congujated by elements of $Gal(\mathbb{Q}(\xi_p)/\mathbb{Q})$. This allows us to construct Heegner points for these Cartan groups. We also show many examples of our construction in cases where they generate the Mordell-Weil group, and relate them to the BSD conjecture. This is based on the work done in http://arxiv.org/abs/1403.7801 .

Joint work with Ariel Pacetti (Universidad de Buenos Aires, Argentina).

Workshop A4
Graph Theory and Combinatorics

Organizers: Yoshiharu Kohayakawa – Gelasio Salazar – Jayme Szwarcfiter

---

A4 - December 11, 14:30 – 15:00

Improved upper bounds on the crossing number, the 2-page crossing number and the rectilinear crossing number of the hypercube

Celina Figueiredo
UFRJ, Brazil
celina@cos.ufrj.br

The n-cube Q_n has as vertices all binary strings of length n, and there is an edge between two strings if and only if they differ in precisely one position. Very little is known about exact values of the crossing number (the minimum number of crossings in a drawing in the plane), the 2-page crossing number, and the rectilinear crossing number for Q_n. We exhibit drawings that are in line with the crossing number conjecture of Erdős and Guy. We also exhibit 2-page and rectilinear drawings, surprisingly with the same number of crossings, despite being completely different constructions.

Joint work with Luerbio Faria (State University of Rio de Janeiro, Brazil), Ondrej Sýkora (Loughborough University, UK), Imrich Vrt'o (Slovak Academy of Sciences, Slovak Republic) and Bruce Richter (University of Waterloo, Canada).

A4 - December 11, 15:00 – 15:30

On the directed cycle double cover conjecture

Andrea Jiménez
University of São Paulo, Brazil
apjr.andrea@gmail.com

Given a graph G, let $D(G)$ denote the direct graph obtained from G by replacing each edge of G by a pair of arcs oppositely directed. The famous directed cycle double cover conjecture, formulated by Jaeger, asserts that if G is a graph without bridges, then the set of arcs of $D(G)$ can be partitioned into directed cycles. In this talk we discuss our recent progress towards a proof of Jaeger's conjecture.

Joint work with Martin Loebl (Charles University, Czech Republic).

A4 - December 11, 15:30 – 16:00

Monochromatic path/cycle partitions

Maya Stein
University of Chile, Chile
mstein@dim.uchile.cl

A conjecture of Gyárfás says that given any colouring with r colours of the edges of the complete graph K_n on n vertices, there are r disjoint monochromatic paths that induce a partition of $V(K_n)$. The conjecture is true for $r \leq 3$. Replacing paths with cycles, it is known that in general, the number of cycles needed is greater than r, but can be bounded by a function of r. (Here, single vertices/edges count as cycles.) For $r = 2$, it is known that 2 paths/cycles suffice.

This talk gives an overview on the history of the problem. We then describe some recent results for bipartite and multipartite graphs, with fixed values of r. We also study variants of the problem for r-local colourings, and for r-mean colourings. The results mentioned are joint work with Conlon, with Lang, with Lang and Schaudt, and with Schaudt, respectively.

A4 - December 11, 16:00 – 16:30

On Connected Identifying Codes for Infinite Lattices

Victor Campos
ParGO - Universidade Federal do Ceará, Brazil
campos@lia.ufc.br

An identifying code in a graph G is a set C of vertices of G such that the closed neighbourhood of every vertex contains a unique and non-empty subset of C. We say that C is a connected identifying code if $G[C]$ is connected. We prove that if a finite graph G on n vertices has maximum degree Δ, then any connected identifying code C satisfies $|C| \geq \frac{2n-2}{\Delta+1}$. We also show this bound is best possible and that the coefficient of n cannot be improved for Δ-regular graphs. We also show that the minimum density of connected identifying codes for the infinite triangular, hexagonal and square lattices are $\frac{1}{3}$, $\frac{1}{2}$ and $\frac{2}{5}$, respectively.

Joint work with Fabrício Benevides (ParGO - Universidade Federal do Ceará), Mitre Dourado (Universidade Federal do Rio de Janeiro), Rudini Sampaio (ParGO - Universidade Federal do Ceará) and Ana Silva (ParGO - Universidade Federal do Ceará).

A4 - December 11, 17:00 – 17:30

Toughness and Kronecker product of graphs

Daniel A Jaume
Universidad Nacional de San Luis, Argentina
djaume@unsl.edu.ar

When investigating the vulnerability of a communication network (graph) to disruption, one may want to find the answer of the following questions (there may be others)

What is the number of elements that are not functioning?

What is the number of remaining connected subnetworks (graphs)?

What is the size of a largest remaining group within mutual communication can still occur (the size of a largest remaining connected component)?

Many graph theoretical parameters such as connectivity, toughness, scattering number, integrity, tenacity, rupture degree, and their edge-analogues, have been defined to obtain the answers to these questions. These parameters have been used to measure the vulnerability of a graph (network)

For most of these parameters, the corresponding computing problems are NP-hard. In this work we generalized a result of Mamut and Vumar, for the toughness of Kronecker product of graphs (Vertex vulnerability parameters of Kronecker products of complete graphs, Information Processing letters 106 (2008) 258-262):

Theorem: Let G be a graph with $t(G) \geq n \geq 3$, then $t(G \times K_n) = n - 1$

Joint work with Adrián Pastine (Michigan Technological University).

A4 - December 11, 18:00 – 18:30

Transversals to the convex hulls of k-sets

Luis Montejano
National University of Mexico, Juriquyilla, Mexico
luismontej@gmail.com

What is the maximum positive integer n such that every set of n points in $\mathbb{R}^d$ has the property that the convex hulls of all k-sets have a transversal $(d - \lambda)$-plane? In this paper, we investigate this and closely related questions. We define a special Kneser hypergraph and by using some topological results and the well-known λ-Helly property, we relate our question with the chromatic number of the Kneser hypergraph, and we establish a connection $(\lambda = 1)$ with so called Kneser's conjecture, first proved by Lovász. This problem is all connected with Gale embeddings, the discrete version of Rado's Problem, and with cyclic polytopes.

Joint work with J. L. Arocha, J. Bracho, J. Chaperon, Leo Martínez, J. Ramirez-Alfonsin and L. P. Montejano.

A4 - December 11, 18:30 – 19:00

The Li-Yau inequality and the geometry of graphs

Paul Horn
University of Denver, United States
paul.horn@du.edu

Harnack inequalities relate the maximum and minimum values of eigenfunctions or positive solutions to the heat equation. These are classical in the manifold setting, and versions are also known for functions on graphs. It is known that a graph satsifying a so-called parabolic Harnack inequality is equivalent to the graph satisfying certain (hard to check) geometric conditions. In the non-negatively curved manifold case, the Li-Yau inequality is a stronger (local) gradient estimate which implies the (global) Harnack inequality. In this talk we describe a similar gradient estimate for graphs.

Along the way, we discuss the issue of defining curvature for graphs and some of the difficulties that arise when transferring a continuous result into a discrete setting along with some additional results on graphs.

Joint work with Frank Bauer (Harvard University), Gabor Lippner (Northeastern University), Yong Lin (Renmin University), Dan Mangoubi (Hebrew University) and Shing-Tung Yau (Harvard University).

A4 - December 12, 14:35 – 15:25

Homomorphisms, Ramsey Theory and Limits

Jaroslav Nešetřil

Charles University, Czech Republic
nesetril@kam.mff.cuni.cz

We present three results from three areas (suggested by the title) which, at least at first glance, have little in common. We shall try to convince the audience of the opposite. Some of the key words: sparse-dense dichotomy, Ramsey classes, structural limits.

A4 - December 12, 16:00 – 16:30

On the Number of Perfect Matchings in Graphs

Marcelo Carvalho
Universidade Federal de Mato Grosso do Sul, Brasil
mhc@facom.ufms.br

In this talk, we survey the main results on the problem of determining the number of perfect matchings in graphs. This problem has been relatively well studied, specially for cubic graphs, but not much is known in the general case. We shall present some new results in the general case and some challenging problems.

Joint work with Cláudio L. Lucchesi (Universidade Federal de Mato Grosso do Sul, Brasil) and U. S. R. Murty (University of Waterloo, Canada).

A4 - December 12, 17:00 – 17:30

Forbidden induced subgraph characterizations of subclasses and variations of perfect graphs

Guillermo Durán
University of Buenos Aires, Argentina
gduran@dm.uba.ar

A graph is perfect if the chromatic number is equal to the clique number for every induced subgraph of the graph. Perfect graphs were defined by Berge in the sixties.

In this survey we present known results about partial characterizations by forbidden induced subgraphs of different graph classes related to perfect graphs.

We analyze a variation of perfect graphs, clique-perfect graphs, and three subclasses of perfect graphs, coordinated graphs, balanced graphs, and neighborhood perfect graphs.

A4 - December 12, 17:30 – 18:00

Characterizing and Recognizing Normal Helly Circular-Arc Graphs

Luciano Grippo
Instituto de Ciencias, Universidad Nacional de General Sarmiento, Argentina
lgrippo@ungs.edu.ar

In this work, we study the intersection graphs of finite sets of arcs on a circle no three of which cover the circle, known as normal Helly circular-arc graphs. Those circular-arc graphs which are minimal forbidden induced subgraphs for the class of normal Helly circular-arc graphs were identified by Lin, Soulignac, and Szwarcfiter, who also posed the problems of determining the remaining minimal forbidden induced subgraphs and finding a direct recognition algorithm. In this work, we solve their problems, obtaining the complete list of minimal forbidden induced subgraphs for the class of normal Helly circular-arc graphs, and presenting a direct recognition algorithm which also finds, in linear time, when the input is a normal Helly circular-arc graph, a minimal forbidden induced subgraph as certificate.

Joint work with Yixin Cao (Department of Computing, Hong Kong Polytechnic University) and Martín Darío Safe (Instituto de Ciencias, Universidad Nacional de General Sarmiento).

A4 - December 12, 18:00 – 18:30

Algorithms and complexity of graph convexity problems

Vinícius dos Santos
Centro Federal de Educação Tecnológica de Minas Gerais, Brazil
viniciussantos@decom.cefetmg.br

Several results concerning convex sets, originally studied on the Euclidean space, have been generalized for abstract convexities. In particular, the results of Carathéodory, Helly and Radon are among the most famous and well-studied. In the last forty years, graph convexities have been studied and graph analogues of classical results have been obtained. More recently, computational aspects have been the focus of many papers of this area.

In this talk we highlight some algorithmic and complexity results of convexity parameters on graphs, discuss some relations with conversion problems and present some open problems.

Joint work with Jayme L. Szwarcfiter (Universidade Federal do Rio de Janeiro, Brazil).

A4 - December 13, 14:30 – 15:00

On the applications of counting independent sets in hypergraphs

Jozsef Balogh
UIUC, USA
jobal@math.uiuc.edu

Recently, Balogh-Morris-Samotij and Saxton-Thomason developed a method of counting independent sets in hypergraphs. During the talk, I show a recent application of the method; solving the following Erdős problem: What is the number of maximal triangle-free graphs?

If there is some extra time in the talk, I will survey some other recent applications.

These applications are partly joint with Das, Delcourt, Liu, Mycroft, Petrickova, Sharifzadeh and Treglown.

A4 - December 13, 15:00 – 15:30

An Erdős-Lovász-Spencer Theorem for permutations and its consequences for parameter testing

Carlos Hoppen
Universidade Federal do Rio Grande do Sul, Brazil
choppen@ufrgs.br

The classical theorem of Erdős, Lovász and Spencer [Strong independence of graphcopy functions, Graph Theory and Related Topics, Academic Press (1979), 165–172] asserts that the densities of connected subgraphs in large graphs are independent. We prove an analogue of this theorem for permutations and apply the methods used in its proof to give an example of a permutation parameter that is both bounded and testable, but not finitely forcible.

Joint work with Roman Glebov (ETH Zürich, Switzerland), Tereza Klimošová (University of Warwick, United Kingdom), Yoshiharu Kohayakawa (Universidade de São Paulo, Brazil), Daniel Král (University of Warwick, United Kingdom) and Hong Liu (University of Illinois at Urbana-Champaign, United States).

A4 - December 13, 15:30 – 16:00

Constructing covering arrays from m-sequences

Daniel Panario
Carleton University, Canada
daniel@math.carleton.ca

Let q be a prime power and $\mathbb{F}_q$ the finite field of q elements. A q-ary m-sequence is a periodic sequence of elements from $\mathbb{F}_q$, which is generated by a linear recurrence relation of order n, and has maximal period $q^n - 1$. These sequences play a crucial role in a wide variety of communications and cryptographic applications.

A covering array $CA(N;t;k;v)$ is a $N \times k$ array with entries from an alphabet of size v, with the property that any $N \times t$ sub-array has at least one row equal to every possible t-tuple. Covering arrays are used in applications such as software and hardware testing. It is crucial for such applications to find covering arrays $CA(N;t;k;v)$ with the smallest N possible, for given t, k, v.

There are various algebraic and combinatorial constructions, as well as computer generation methods for covering arrays. Although constructions using m-sequences exist in the literature, these are a few and, until recently, only focused on similar combinatorial objects ("orthogonal arrays" that are covering arrays with more restrictions). Moura, Raaphorst and Stevens (2013) give a construction for covering arrays of strength 3 using m-sequences, which are the best known for many cases. For any prime power q, they give a covering array of strength $t = 3$ with $k = q^2 + q + 1$ columns over $v = q$ symbols that has size $N = 2q^3 - 1$ (number of rows).

In this talk we present an extension of this construction to strengths greater than or equal to 4. The construction is based on Linear Feedback Shift Register (LFSR) sequences constructed using primitive polynomials over finite fields. We have also developed a backtracking algorithm that yields new covering arrays of strength 4. For certain parameters, these are either the best, or close to being the best known when compared to the best known covering array tables kept by Colbourn. Furthermore, our findings show interesting connections with finite geometry.

Joint work with Lucia Moura (University of Ottawa, Canada), Brett Stevens (Carleton University, Canada) and Georgios Tzanakis (Carleton University, Canada).

A4 - December 13, 16:00 – 16:30

A unified approach to linear probing hashing

Alfredo Viola
Universidad de la República, Uruguay
viola@fing.edu.uy

We give a unified analysis of linear probing hashing with a general bucket size. We use both a combinatorial approach, giving exact formulas for generating functions, and a probabilistic approach, giving simple derivations of asymptotic results. Both approaches complement nicely, and give a good insight in the relation between linear probing and random walks. The Poisson Transform links in a natural way both approaches. A key methodological contribution, at the core of Analytic Combinatorics, is the use of the symbolic method (based on q-calculus) to directly derive the generating functions to analyze.

Joint work with Svante Janson (Uppsala University, Sweden).

A4 - December 13, 17:05 – 17:55

Extremal combinatorics in random discrete structures

Mathias Schacht
Universität Hamburg, Germany
schacht@math.uni-hamburg.de

We survey on recent results at the intersection of extremal combinatorics and random graph theory. More precisely, we consider thresholds for extremal properties of random discrete structures. Among other problems, we shall discuss the threshold for Szemerédi's theorem on arithmetic progressions in random subsets of the integers and the threshold for Turán-type problems for random graphs and hypergraphs, which were obtained independently by Conlon and Gowers and by the speaker. Furthermore, we discuss recent general results on independent sets in hypergraphs by Balogh, Morris and Samotij and by Thomason and Saxton, which led to new proofs of these results and have already had many other applications in the area.

A4 - December 13, 18:05 – 18:55

Towards a Broader View of Theory of Computing – Part 2

Narendra Karmarkar
Indian Institute of Technology, Bombay, India
narendrakarmarkar@yahoo.com

Beginning with the projectively invariant method for linear programming, interior point methods have led to powerful algorithms for many difficult computing problems, in combinatorial optimization, logic, number theory and non-convex optimization. Algorithms for convex optimization benefitted from many pre-established ideas from classical mathematics, but non-convex problems require new concepts. This three part series outlines some of these concepts – computational models based on the concept of the continuum, algorithms invariant with respect to transformations on co-ordinate representation-projective, bi-rational, and bi-holomorphic transformations, extended proof systems for more efficient certificates of optimality, extensions of Grassmann's extension theory, efficient evaluation methods for the effect of exponential number of constraints, theory of connected sets based on graded connectivity, theory of curved spaces adapted to the problem data, and concept of "relatively" algebraic sets in curved space.

Topics for part 2 - In this session, we show how algorithms based on continuum computing can obtain exponential speed-up over those based on Turing machines by considering concrete examples of finding maximum independent set in a graph and the satisfiability problem. We review the concept of graph minors based on parity-respecting homeomorphisms. Although there can be exponential number of subgraphs homeomorphic to a given minor, we show how their combined effect can be computed in polynomial number of operations in the continuum approach.

We extend the concept of graph minors to sub-formulas in a satisfiability problem. As shown by Chvátal and Szemerédi, almost all instances of the problem for certain range of parameter require exponential time for resolution. We identify sub-formulas, which we call “mobius cycles”, as the culprits causing exponential behavior, and show that their combined effect can be computed in polynomial number of operations in the continuum approach. The method can be thought of as generalizing the generating functions used in enumerative combinatorics to continuum based algorithms. Objective function for these problems is treated by non-convex optimization methods covered in part 3.

Ability of algebroid functions to efficiently encode and process information regarding exponential number of combinatorial substructures should not come as a surprise, given that the most famous meromorphic function – the Riemann Zeta function – is able to carry information regarding the infinite sequence of primes.

A4 - Poster

Characterizations of (k)-interval graphs

Florencia Fernández Slezak
Instituto de Cálculo, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Argentina
ffslezak@gmail.com

Keywords: unit interval graphs, intersection graphs.

A graph G is an interval graph iff its vertices can be put in a one to one correspondence with a family of intervals on the real line such that two vertices in G are adjacent if and only if their corresponding intervals intersect. Such a family of intervals is called an interval model for G [1, 2].

If there is an interval model for graph G such that the intervals are of unit length, then it is called a *unit interval model* and G is a *unit interval graph*. A theorem was proved which establishes that G is a unit interval graph if and only if G is an interval graph containing no induced claw, that is, $K_{1,3}$ [6, 7].

A unit interval graph is an interval graph which admits a representation where all elements have the same length. It is known that a model with intervals of the same length and integer extremes can be built in $\mathcal{O}(n+m)$ time to represent a given unit interval graph [4, 5]. Let $k \in \mathbb{N}_0$, we define G as a (k)-interval graph iff G is a unit interval graph which admits a model of open intervals of length k and integer extremes. Likewise, we define the $[k]$-interval graphs as those unit interval graphs which admit a model with the same characteristics but with closed intervals.

In this work, we present a structural characterization of the simplicial vertices in a unit interval graph without twins, which leads to a characterization of the (k)-interval graphs. We study the structure of these graphs, finding forbidden induced subgraphs, thus a theorem fully characterising this class is posed, finding the least k such that G is a (k)-interval graph.

Moreover, the (k)-interval graphs will be studied as induced subgraphs of power of paths, continuing with the results obtained by Lin, Rautenbach, Soulignac and Szwarcfiter [3].

Furthermore, an equivalence between the $[k]$-interval and the (k)-interval graphs will be set, which allows us to use the obtained results for open intervals as well as closed intervals.

Finally, we will present open problems we are studying at the moment.

References:

[1] Golumbic, M. C., Algorithmic graph theory and perfect graphs, Academic Press New York (1980)

[2] Lekkerkerker, C. and Boland, D., Representation of finite graphs by a set of intervals on the real line, FundMath, 51, 45–64 (1962).

[3] Lin, M. C., Rautenbach, D., Soulignac, F., Szwarcfiter, J. L., Powers of Cycles, Powers of Paths, and Distance Graphs, Discrete Applied Mathematics, 159, 621–627 (2011).

[4] Lin, M. C., Soulignac, F. J., Szwarcfiter, J. L., Short Models for Unit Interval Graphs, Electronic Notes in Discrete Mathematics, Amsterdam, 35, 247–255 (2009).

[5] Corneil, D. G., Kim, H., Natarajan, S., Olariu, S., Sprague, A. P., Simple linear time recognition of unit interval graphs, Information Processing Letters, 55, 99–104 (1995).

[6] Roberts, F. S., Indifference graphs, in: Proof Techniques in Graph Theory, Proc. Second Ann Arbor Graph Theory Conf., Ann Arbor, Mich., Academic Press, New York, 139–146 (1968).

[7] Gardi, F., The Roberts characterization of proper and unit interval graphs, Discrete Mathematics, 22(307), 2906 – 2908 (1968).

Joint work with Guillermo Durán (Universidad de Buenos Aires and Universidad de Chile), Luciano Grippo (Universidad Nacional de General Sarmiento), Fabiano Oliveira (Universidade do Estado do Rio de Janeiro) and Jayme Szwarcfiter (Universidade Federal do Rio de Janeiro).

A4 - Poster

Covering edge multicolorings of complete graphs with monochromatic connected components

Sebastián Bustamante
Universidad de Chile, Chile
sbustamante@dim.uchile.cl

Let K_n denote the complete graph on vertex set $V = [n]$. An r-edge multicoloring is a coloring of the edges with any number of colors in $[r]$. If every edge is colored by a constant number k of colors, we say that the r-edge multicoloring is k-regular. The connected components of the monochromatic graphs given by $G_i = (V_i, E_i)$, with $e \in E_i$ if the edge e has the color i, for $i \in [r]$, are called monochromatic connected components. For $r > k$, let $f(r,k)$ the smallest number such that, for any k-regular r-edge multicoloring of a complete graph, it is possible to cover the complete graph by $f(r,k)$ monochromatic connected components. A reformulation of an important special case of Ryser's conjecture states that $f(r,1) = r-1$ for all r. This conjecture is known to be true for $r \leq 5$. We prove, for $k \geq 2$, several exact values and bounds for $f(r,k)$, and a related result of independent interest, as the maximum number of vertices for a vertex minimal r-edge multicoloring of a complete graph coverable by three, but no by two, monochromatic connected components.

Joint work with Maya Stein (Universidad de Chile, Chile).

A4 - Poster

Alignments of a Pair of Graphs

Cesim Erten
Kaidr Has University, Turkey
cesimerten@gmail.com

The graph alignment problem has important applications in biological network alignment. In the case of protein-protein interaction networks for instance, undirected graphs G_1, G_2 correspond to PPI networks from a pair of species, where each of the vertex sets V_1, V_2 represent the sets of proteins, and E_1, E_2 represent respectively the sets of known protein interactions pertaining to the networks of species under consideration. The informal goal is to find a one-to-one mapping between V_1, V_2 that maximizes the conservation of interactions between the pairs of mappings. A specific version of the problem reduces the size of the problem by restricting the output alignment mappings to those chosen among certain subsets of protein mappings. The "constrained graph alignment" is considered a graph theoretical generalization of this biological network alignment problem.

Let $G_1 = (V_1, E_1), G_2 = (V_2, E_2)$ be undirected graphs and S be a bipartite graph defined on (V_1, V_2) as the partition. For S, assume the degree of each vertex from the part V_i is at most m_i, for $i = 1, 2$. A *legal alignment* A is a matching of S. Let u_1u_2, v_1v_2 be a pair of edges of A such that $u_1, v_1 \in V_1$ and $u_2, v_2 \in V_2$. This pair of edges gives rise to a *conserved edge* if and only if $u_1v_1 \in E_1$ and $u_2v_2 \in E_2$. The constrained alignment problem is that of finding a legal alignment that maximizes the number of conserved edges. A 4-cycle x denoted with $c_4(x) = abcd$ is a cycle $a - b - c - d - a$ where $ab \in E_1$, $cd \in E_2$ and $ad, bc \in S$. We say that two c_4s "conflict" if their edges from S cannot coexist in any legal alignment; the c_4s contain at least one pair of S edges which cannot coexist in a matching of S. For a given $\prec G\_1, G\_2, S \succ$ instance, we construct a "conflict graph", $\mathcal{C}$, as follows: For each c_4 create a vertex in $\mathcal{C}$ and for each pair of conflicting c_4s create an edge between their respective vertices in $\mathcal{C}$. With this construction, the constrained alignment problem obviously reduces to the maximum independent set problem. Let $\mathcal{M}$ be and independent set of the conflict graph $\mathcal{C}$. The set of S edges included in the c_4s corresponding to the vertices in $\mathcal{M}$ constitute an optimum solution of the constrained alignment instance $\prec G\_1, G\_2, S \succ$, that is a legal alignment with the maximum number of conserved edges, if and only if $\mathcal{M}$ is a maximum independent set of $\mathcal{C}$. The following results apply to the case where $m_2 = 1$, that is each vertex of G_2 can be mapped to a single vertex of G_1. Fertin et al. studied the constrained alignment problem under the same setting, that is $m_2 = 1$. Although they also define a conflict graph and provide a fixed-parameter tractability result based on a simple argument regarding the degrees of the conflict graphs, they do not provide any further structural properties. In what follows we extend their results and provide several graph-theoretic properties of conflict graphs arising from possible constrained alignment instances under various restrictions. Such properties are useful in applying relevant maximum independent set results. We skip all the proofs due to space restrictions.

Denote a k-cycle with C_k and denote the complement of G with $\overline{G}$. A "weakly triangulated" graph contains neither a C_k nor $\overline{C_k}$, for $k \geq 5$. The following theorem in connection with the "strong perfect graph theorem" of Chudnovsky et al. implies that the conflict graphs under the considered setting are perfect.

Theorem: If G_1 is acyclic and $m_2 = 1$ then $\mathcal{C}$ is weakly triangulated.

Below we present graph theoretic properties of conflict graphs removing any restriction on G_1 or G_2, but imposing a further constraint on m_1.

Theorem: If $m_1 = 2$ and $m_2 = 1$, the wheel $W_k, k \geq 5$ is not an induced subgraph of a conflict graph.

We now generalize the previous results by providing further structural properties of conflict graphs for the more general case where m_1 can be any positive integer constant. We first present our result analogous to the previous theorem.

Theorem: If $m_2 = 1$, W_k is not an induced subgraph of $\mathcal{C}$, for $k \geq 7$.

Next we present our results regarding the existence of cliques as subgraphs of conflict graphs for any m_1.

Theorem: If $m_2 = 1$, the maximum size of any clique in $\mathcal{C}$ is m_1^2.

The following result characterizes the conflict graphs that do not contain certain claws as induced subgraphs. A d-claw is an induced subgraph of an undirected graph, that consists of an independent set of d vertices, called *talons*, and the *center* vertex that is adjacent to all vertices in this set. Let $\Delta_{min} = min(\Delta_1, \Delta_2)$, where Δ_1, Δ_2 are the maximum degrees of G_1 and G_2 respectively.

Theorem: If $m_2 = 1$, $(2\Delta_{min} + 2)$-claw is not an induced subgraph of $\mathcal{C}$.

Joint work with Ferhat Alkan (University of Copenhagen, Denmark), Turker Biyikoglu (Izmir Institute of Technology, Turkey) and Marc Demange (RMIT University, Australia).

A4 - Poster

Partitioning 3-edge-coloured complete bipartite graphs into monochromatic cycles

Richard Lang
Universidad de Chile, Chile
rlang@dim.uchile.cl

We show that any colouring with three colours of the edges of the complete bipartite graph $K_{n,n}$ contains 18 vertex-disjoint monochromatic cycles which together cover all vertices. The minimum number of cycles needed for such a covering is 5, and we show that this lower bound is asymptotically true.

Joint work with Maya Stein (Universidad de Chile, Chile) and Oliver Schaudt (Universität zu Köln, Germany).

A4 - Poster

Local EPT graphs on bounded degree trees

María Pía Mazzoleni
Universidad Nacional de La Plata, CONICET, Argentina
pia@mate.unlp.edu.ar

A graph G is called an EPT graph if it is the edge intersection graph of a family of paths in a tree. An EPT representation of G is a pair $\langle \mathcal{P}, T\rangle$ where $\mathcal{P}$ is a family $(P_v)_{v\in V(G)}$ of subpaths of the host tree T satisfying that two vertices v and v' of G are adjacent if and only if P_v and $P_{v'}$ have at least two vertices (one edge) in common. When the maximum degree of the host tree T is at most h, the EPT representation of G is called an (h,2,2)-representation of G. The class of graphs which admit an (h,2,2)-representation is denoted by [h,2,2].

Notice that the class of EPT graphs is the union of the classes [h,2,2] for $h \geq 2$. It was proved that the recognition of EPT graphs is an NP-complete problem.

The EPT graphs are used in network applications, where the problem of scheduling undirected calls in a tree network is equivalent to the problem of coloring an EPT graph.

In this paper, we examine the local structure of paths passing through a given vertex of a host tree which has maximum degree h, and show these locally EPT graphs are equivalent to the line graphs of certain graphs.

Definition: Let $\langle \mathcal{P}, T\rangle$ be an EPT representation of a graph G. A pie of size n is a star subgraph of T with central vertex q and neighbors q_1,...,q_n such that each "slice" $q_i\, q\, q_{i+1}$ for $1 \leq i \leq n$ is contained in a different member of $\mathcal{P}$; addition is assumed to be module n.

Let $\langle \mathcal{P}, T\rangle$ be an EPT representation of a graph G. It was proved that if G contains a chordless cycle of length $n \geq 4$, then $\langle \mathcal{P}, T\rangle$ contains a pie of size n.

Definition: We say that $\langle \mathcal{P}, T\rangle$ is a local EPT representation of G if it is an EPT representation where all the paths of $\mathcal{P}$ share a common vertex of T.

We call G a local EPT graph if it has a local EPT representation.

Let $h \geq 5$, we say that G belongs to the class [h,2,2] local if and only if G has a local EPT representation in a host tree T with maximum degree h.

In this work, we characterize the graphs which belongs to the class [h,2,2] local.

Definition: Let G be a connected graph. We say that $v \in V(G)$ is a cut vertex of G if G-v has at least two connected components.

Theorem 1: Let $h \geq 5$. If $G \in [h, 2, 2]$ local and $G \notin [h-1, 2, 2]$ then G has no cut vertices.

We show that this special subclass of EPT graphs is equivalent to the class of line graphs of certain graphs which have certain properties.

Definition: Let H be a graph, the line graph of H, noted by L(H), has vertices corresponding to the edges of H with two vertices adjacent in L(H) if their corresponding edges of H share an endpoint.

Definition: We say that two vertices u,v of G are adjacent dominated vertices if $uv \in E(G)$ and $N_G(u) \subseteq N_G(v)$ or $N_G(v) \subseteq N_G(u)$.

Theorem 2: Let $h \geq 5$. $G \in [h, 2, 2]$ local, $G \notin [h-1, 2, 2]$ and G without adjacent dominated vertices if and only if G=L(H) with H a graph such that:

(i) —V(H)—=h; (ii) H has no vertices of degree 1; (iii) H is simple; (iv) H has no adjacent dominated vertices; (v) H has a cycle C_n, with $4 \leq n \leq h$; and every vertex of $H - C_n$ is in some path between two different vertices of C_n.

Conjecture: Let $h \geq 5$. If $G \in [h, 2, 2]$, $G \notin [h-1, 2, 2]$ but $G - v \in [h-1, 2, 2]$, for all $v \in V(G)$, then $G \in [h, 2, 2]$ local.

Joint work with Liliana Alcón (Universidad Nacional de La Plata, Argentina) and Marisa Gutierrez (Universidad Nacional de La Plata, CONICET, Argentina).

A4 - Poster

Methods for reliability analysis of diameter constrained networks

Denis Migov
Institute of Computational Mathematics and Mathematical Geophysics of SB RAS, Russia
mdinka@rav.sscc.ru

Networks where the edges are a subject to random failures are studied in the present article. As a rule networks with unreliable elements are modeled by a probabilistic graph in which an operational probability is associated with every element (a node or an edge). The most common reliability measure of such networks is the probability that all the terminal nodes in a network can keep connected together, given the reliability of each network node and edge. The analysis of the network reliability has been a subject of considerable research [1, 2, 3]. Today we have a lot of different methods for network reliability calculation and evaluation. These methods may be useful during network topology design and optimization.

In practice it often needs not only the existence of a path between each pair of nodes, but the existence of a path passing via a limited number of communication links. Thus, we arrive to a different reliability measure. The diameter constrained network reliability (DCNR) is a probability that every two nodes from a given set of terminals are connected with a path of length less or equal to a given integer. By the length of a path we understand the number of edges in this path.

This reliability measure was introduced in [4] and studied in more detail in [5, 6]. The problem of computing this measure in general is known to be NP-hard, just like the problem of computing the probability of network connectivity. Now the complexity of DCNR calculation is completely studied [6].

The new approach in reliability calculation (without diameter constraint) was introduced in [2]: cumulative update of lower and upper bounds of network reliability for faster feasibility decision. This method allows to decide the feasibility of a given network without performing exhaustive calculation. The approach was further developed with the help of network decomposition [3]. We propose the method for cumulative updating of lower and upper bounds of diameter constrained network reliability. This method allows us to make a decision about the network reliability (or unreliability) with diameter constraint with respect to a given threshold without performing the exhaustive calculation.

One of the main reasons, which make the calculation of diameter constrained network reliability much more complicated in comparison with other network reliability measures, is the lack of methods of recursion quantity decrease. For example, for k-terminal network reliability calculation we may use a lot of network reduction and decomposition methods. We propose to use the decomposition methods and some other techniques for DCNR calculation. Let us demonstrate one of these methods.

Suppose that graph G includes cutnode x dividing the graph in two G_1 and G_2 subgraphs. Let us consider 2-terminal case. Let terminals s and t belong to G_1 and G_2 respectively and none of them is x. DNCR is defined as the probability that nodes s and t are connected with diameter constrain d in a graph G. We denote it by $R^d_{s,t}(G)$. By $R^{\bar{d}}_{s,t}(G)$ we denote a probability that nodes s and t are connected in G and a length of shortest path between them is equal to d. By d_i we denote the distance between s and x or y in subgraph G_i. We obtained how $R^d_{s,t}(G)$ is expressed in terms of $R^i_{s,x}(G_1)$, $R^{\bar{i}}_{s,x}(G_1)$, $R^i_{x,t}(G_2)$, and $R^{\bar{i}}_{x,t}(G_2)$:

$$R^d\_s,t(G) = \sum^{d-d\_2} \_i = d\_1R^{\bar{i}}\_s,x(G\_1)R^{d-i}\_x,t(G\_2) = \sum^{d-d\_1} \_i = d\_2R^{d-i}\_s,x(G\_1)R^{\bar{i}}\_x,t(G\_2). \tag{1}$$

Also we obtained some other effective techniques for DCNR calculation, including the formula that lets performing decomposition for the computation of diameter constrained 2-terminal reliability of a network that contains 2-node cuts. While the calculation of DCNR known to be a NP-hard problem, the proposed methods allow to considerably reduce the calculation time.

This work was supported by Russian Foundation for Basic Research under Grant 14-07-31069 and by Novosibirsk city administration under Grant 39-14.

References

1. Ball M.O. Computational complexity of network reliability analysis: An overview // IEEE Transactions on Reliability. Vol. 35, 1986, p. 230-239.

2. Won J.-M., Karray F. Cumulative Update of All-Terminal Reliability for Faster Feasibility Decision // IEEE Transactions on Reliability. Vol. 59, issue 3, 2010, 551-562.

3. Rodionov A.S., Migov D.A., Rodionova O.K. Improvements in the Efficiency of Cumulative Updating of All-Terminal Network Reliability // IEEE Transactions on Reliability. Vol. 61, issue 2, 2012, p. 460-465.

4. Cancela H., Petingi L. Diameter Constrained Network Reliability: Exact Evaluation by Factorization and Bounds // Proc. Int. Conf. on Industrial Logistics. Japan, 2001, p. 359-356.

5. Migov D.A. Computing Diameter Constrained Reliability of a Network with Junction Points // Automation and Remote Control. Vol. 72, issue 7, 2011, p. 1415-1419.

6. Eduardo Canale, Hector Cancela, Franco Robledo, Gerardo Rubino and Pablo Sartor. On Computing the 2-diameter-constrained K-reliability of Networks // International Transactions in Operational Research. Vol. 20, issue 1, January 2013, p. 49-58.

A4 - Poster

Counting k-uniform linear hypergraphs in sparse pseudorandom hypergraphs

Guilherme Oliveira Mota
USP - University of São Paulo, Brazil
mota@ime.usp.br

We give a counting lemma for the number of copies of linear k-uniform connector-free hypergraphs (connector is a generalization of triangle for hypergraphs) that are contained in some sparse hypergraphs G. Let H be a linear k-uniform connector-free hypergraph and let G be a k-uniform hypergraph with n vertices. Set $d_H = \max\{\delta(J)\colon J \subset H\}$ and $D_H = \min\{kd_H, \Delta(H)\}$. We proved that if the vertices of G do not have large degree, small families of $(k-1)$-element sets of $V(G)$ do not have large common neighbourhood and most of the pairs of sets in $\binom{V(G)}{k-1}$ have the 'right' number of common neighbours, then the number of embeddings of H in G is $(1+o(1))n^{|V(H)|}p^{|E(H)|}$, given that $p \gg n^{-1/D_H}$ and $|E(G)| = \binom{n}{k}p$. This generalizes a result by Kohayakawa, Rödl and Sissokho [Embedding graphs with bounded degree in sparse pseudo-random graphs, Israel J. Math. 139 (2004), 93-137], who proved that, for p as above, this result holds for graphs, where H is a triangle-free graph.

Joint work with Yoshiharu Kohayakawa, Mathias Schacht and Anusch Taraz.

A4 - Poster

Stability and Ramsey numbers for cycles and wheels

Nicolás Sanhueza
Universidad de Chile, Chile
nicolas@sanhueza.net

We study the structure of red-blue edge-colorings of a complete graph, in such a way that certain graphs don't appear as monochromatic subgraphs. More concretely, we consider the case of an odd positive integer n, and forbidden monochromatic graphs given by a red n-cycle C_n and a blue n-wheel $W_n = C_n + K_1$.

Our main result is that if complete graph G of appropriate size has a red-blue edge-coloring of its edges in a way such that C_n is not a red subgraph of G nor W_n is a blue subgraph of G; then deleting at most two vertices we obtain a partition of the vertices of G in three sets such that the edges with both ends in the same element of the partition are colored red, and blue otherwise.

We can see this result as a generalization of results previously shown by Nikiforov and Schelp, in the case where the forbidden monochromatic subgraphs are odd cycles.

As a secondary result of our proof, we obtain two bounds for the Ramsey number of $r(C_{2k+1}, W_{2k+1})$: one is tighter for small values of k, and the other is better in the asymptotic case. The exact values for $r(C_{2k+1}, W_{2k+1})$ are presently an open problem. Our bounds are a rough approximation to the conjectured values and show that they are, at least, asymptotically true.

Joint work with Maya Stein (Universidad de Chile, Chile).

A4 - Poster

Linear-time algorithms for neighborhood covering and independence on two superclasses of cographs

Xavier Sebastián Warnes
Universidad de Buenos Aires, Argentina
xwarnes@dc.uba.ar

Given a simple graph G, a set $C \subseteq V(G)$ is a neighborhood cover set if every edge and vertex of G belongs to some $G[v]$ with $v \in C$, where $G[v]$ denotes the subgraph of G induced by the closed neighborhood of the vertex v. Two elements of $E(G) \cup V(G)$ are neighborhood-independent if there is no vertex $v \in V(G)$

such that both elements are in $G[v]$. A set $S \subseteq V(G) \cup E(G)$ is said to be a neighborhood-independent set if every pair of elements of S is neighborhood-independent. Let $\rho_{\mathrm{n}}(G)$ be the size of a minimum neighborhood cover set and $\alpha_{\mathrm{n}}(G)$ of a maximum neighborhood-independent set. Lehel and Tuza defined neighborhood-perfect graphs as those where $\rho_{\mathrm{n}}(G') = \alpha_{\mathrm{n}}(G')$ for every induced subgraph G' of G.

In this work we present linear-time algorithms for finding a minimum neighborhood cover set and a maximum neighborhood-independent set for two superclasses of cographs — namely, P_4-tidy graphs and tree-cographs. We also give linear-time algorithms for recognizing neighborhood-perfect graphs within the same graph classes. These algorithms rely on the structure of the corresponding modular decomposition trees.

Joint work with Guillermo Durán (Universidad de Buenos Aires, Argentina) and Martín Safe (Universidad Nacional de General Sarmiento, Argentina).

Workshop A5
Multiresolution and Adaptivity in Numerical PDEs

Organizers: Annalisa Buffa – Angela Kunoth – Pedro Morín

A5 - December 11, 14:30 – 15:10

A posteriori error estimation and adaptivity in the maximum norm

Alan Demlow
Texas A&M University, USA
demlow@math.tamu.edu

In this talk I will discuss recent progress in constructing adaptive algorithms for controlling errors in the maximum norm. The main focus of the talk will be the construction of robust a posteriori error estimators for singularly perturbed elliptic reaction-diffusion problems. Time permitting, I may also discuss the sharpness of logarithmic factors that typically arise in maximum norm a posteriori estimates.

Joint work with Natalia Kopteva (University of Limerick).

A5 - December 11, 15:35 – 16:25

Weak convergence analysis for stochastic evolution PDEs

Stig Larsson
Chalmers University of Technology and University of Gothenburg, Sweden
stig@chalmers.se

I will review the literature on the weak convergence analysis of numerical methods for stochastic evolution problems driven by noise. Then I will present a new method of proof, which is based on refined Sobolev-Malliavin spaces from the Malliavin calculus. It does not rely on the use of the Kolmogorov equation or the Ito formula and is therefore applicable also to non-Markovian equations, where these are not available. We use it to prove weak convergence of fully discrete approximations of the solution of the semilinear stochastic parabolic evolution equation with additive noise as well as a semilinear stochastic Volterra integro-differential equation.

Joint work with Adam Andersson (Chalmers University of Technology and University of Gothenburg, Sweden), Mihaly Kovacs (University of Otago, New Zealand) and Raphael Kruse (TU Berlin, Germany).

A5 - December 11, 17:00 – 17:20

Existence of p-Moments for the Weak Space-Time Heat Equation with Random Coefficients and Stability of its Petrov-Galerkin Discretization

Christian Mollet
University of Cologne, Germany
cmollet@math.uni-koeln.de

We consider the heat equation in a weak space-time formulation with random right hand side and random spatial operator. The existence and uniqueness of a solution can be proven by the Banach-Necas-Babuska theorem. In this course we allow the spatial operator A to have lower and upper bounds depending on a stochastic parameter ω, i.e., we consider random variables $A_{\min}(\omega)$ and $A_{\max}(\omega)$ as lower and upper bounds. The L_p-regularity of the solution and its connection to the random variables bounding A can be proven.

A similar approach is applied to a full space-time Petrov-Galerkin discretization. The stability of such an approach requires a lower bound for the discrete inf-sup condition independent of the grid spacing. Using similar ideas, we can prove stability when allowing a finer discretization for the test space than for the solution space.

Joint work with Stig Larsson (Chalmers University of Technology) and Matteo Molteni (Chalmers University of Technology).

A5 - December 11, 17:30 – 17:50

Regularity of boundary integral equations in Besov-type spaces based on wavelet expansions

Markus Weimar
Philipps-University Marburg, Germany
weimar@mathematik.uni-marburg.de

We study regularity properties of solutions to operator equations on patchwise smooth manifolds $\partial\Omega$ such as, e.g., boundaries of polyhedral domains $\Omega \subset \mathbb{R}^3$. Using suitable biorthogonal wavelet bases Ψ, we introduce a new class of Besov-type spaces $B^{\alpha}_{\Psi,q}(L_p(\partial\Omega))$ of functions $u\colon \partial\Omega \to \mathbb{C}$. Special attention is paid on the rate of convergence for best n-term wavelet approximation to functions in these scales since this determines the performance of adaptive numerical schemes. We show embeddings of (weighted) Sobolev spaces on $\partial\Omega$ into $B^{\alpha}_{\Psi,\tau}(L_\tau(\partial\Omega))$, $1/\tau = \alpha/2 + 1/2$, which lead us to regularity assertions for the equations under consideration. Finally, we apply our results to a boundary integral equation of the second kind which arises from the double layer ansatz for Dirichlet problems for Laplace's equation in Ω. The talk is based on two recent papers which arose from the DFG-Project "BIOTOP: Adaptive Wavelet and Frame Techniques for Acoustic BEM" (DA 360/19-1):

Dahlke, S. and Weimar, M.: Besov regularity for operator equations on patchwise smooth manifolds. Preprint 2013-03, Fachbereich Mathematik und Informatik, Philipps-Universität Marburg. To appear in J. Found. Comput. Math.

Weimar, M.: Almost diagonal matrices and Besov-type spaces based on wavelet expansions. Preprint 2014-06, Fachbereich Mathematik und Informatik, Philipps-Universität Marburg. Submitted.

Joint work with Stephan Dahlke (Philipps-University Marburg, Germany).

A5 - December 11, 18:00 – 18:40

T-spline adaptivity

Michael Scott
Brigham Young University, U.S.A.
michael.scott@byu.edu

T-splines are a CAD geometry description which overcome the strict four-sided topological constraint inherent in NURBS-based CAD, the current industry standard. An important property of T-splines is the ability to perform local refinement while maintaining exact geometry and smoothness. Analysis-suitable T-splines are a mildly restricted class of T-splines which possess the important mathematical properties of NURBS and the geometric flexibility of T-splines. Analysis-suitable T-splines are now widely used as a basis for isogeometric design and analysis. In this talk, I will focus on analysis-suitable T-spline adaptivity and its application in design and analysis. Analysis-suitable T-splines will be defined and several of the most important mathematical properties of the resulting splines spaces will be discussed. In particular, h, p, and k-adaptivity will be treated for both single-level and multi-level T-spline descriptions. Several challenging applications where T-spline adaptivity is being used will be presented.

A5 - December 12, 14:30 – 15:10

Near-Best hp-Adaptive Approximation

Peter Binev
University of South Carolina, United States
binev@math.sc.edu

The hp-adaptive numerical methods for PDEs combine the domain decomposition with assignment of degrees of freedom at each element of a particular refinement. The main objective of this talk is to introduce a framework that streamlines the process of making adaptive decisions.

We consider domain partitioning based on a fixed binary refinement scheme and a coarse-to-fine routine for making adaptive decisions about the elements to be split and the polynomial orders to be assigned. The problem of finding near-optimal results is managed by using greedy algorithms on binary trees and a modification of the local errors that take into account the local complexity of the adaptive approximation. We prove that the algorithm provides near-best approximation to any given function.

A5 - December 12, 15:30 – 16:10

convergence and optimality of adaptive boundary element methods

Jens Markus Melenk
Vienna University of Technology, Austria
melenk@tuwien.ac.at

We consider the h-version of the boundary element method (BEM) in 2D and 3D on shape regular meshes. We show convergence and prove quasi-optimality of an adaptive BEM (ABEM), taking Symm's integral equation as our model problem for a first kind integral equation. Optimality here means that the algorithm realizes the optimal rate achievable for solutions for an approximation class that is characterized by the best possible decay rate achievable for the error indicator under the mesh refinements allowed (here: newest vertex bisection). The error indicators that drive the adaptive algorithm are of residual type and hark back to [**?**, **?**]. For the FEM on shape regular meshes, similar convergence and optimality results are available, (Stevenson 2007; Cascon, Kreuzer, Nochetto, Siebert 2008). The BEM setting is more involved and requires different mathematical tools since the operators and pertinent norms are non-local. This mandates in particular the use of non-standard inverse estimates for integral operators, which we present in this talk. We will also discuss extensions of the algorithm to account for data approximations and applications to FEM-BEM coupling.

Joint work with Michael Feischl (Vienna University of Technology, Austria), Michael Karkulik (Pontificia Universidad Catolica de Chile, Chile) and Dirk Praetorius (Vienna University of Technology, Austria).

A5 - December 12, 17:05 – 17:55

Adaptive low-rank tensor approximation for high dimensional operator equations

Wolfgang Dahmen
RWTH Aachen, Germany
dahmen@igpm.rwth-aachen.de

Problems in high spatial dimensions are typically subject to the "curse of dimensionality" which roughly means that the computational work, needed to approximate a given function within a desired target accuracy, increases exponentially in the spatial dimension. A possible remedy is to seek problem dependent dictionaries with respect to which the function possesses sparse approximations. Employing linear combinations of particularly adapted rank-one tensors falls into this category. In this talk we highlight some recent developments centering on the adaptive solution of high dimensional operator equations in terms of stable tensor expansions. Some new concepts related to tensor contractions, tensor recompression, coarsening, and rescaling operators are outlined. Some essential issues are addressed that arise in the convergence and complexity analysis but have been largely ignored when working in a fully discrete setting. In particular, when dealing with high-dimensional diffusion problems, a central obstruction is related to the spectral properties of the underlying operator which is an isomorphism only when acting between spaces that are not endowed with tensor product norms. The theoretical results are illustrated by numerical experiments.

Joint work with Markus Bachmayr (RWTH-Aachen, Germany).

A5 - December 12, 18:00 – 18:40

Polynomial-degree-robust a posteriori estimates in a unified setting

Martin Vohralik
INRIA Paris-Rocquencrourt, France
martin.vohralik@inria.fr

We present equilibrated flux a posteriori error estimates in a unified setting for conforming, nonconforming, discontinuous Galerkin, and mixed finite element discretizations of the two-dimensional Poisson problem. Relying on the equilibration by mixed finite element solution of patchwise Neumann problems, the estimates are guaranteed, locally computable, locally efficient, and robust with respect to polynomial degree. Maximal local overestimation is guaranteed as well. Numerical experiments suggest asymptotic exactness for the incomplete interior penalty discontinuous Galerkin scheme.

Joint work with Alexandre Ern (CERMICS, Universite Paris-Est, France).

A5 - December 13, 14:30 – 15:10

Adaptive Wavelet Boundary Element Methods

Helmut Harbrecht

University of Basel, Switzerland
helmut.harbrecht@unibas.ch

This talk is concerned with developing numerical techniques for the adaptive application of global operators of potential type in wavelet coordinates. This is a core ingredient for a new type of adaptive solvers that has so far been explored primarily for partial differential equations. We shall show how to realize asymptotically optimal complexity in the present context of global operators. Asymptotically optimal means here that any target accuracy can be achieved at a computational expense that stays proportional to the number of degrees of freedom (within the setting determined by an underlying wavelet basis) that would ideally be necessary for realizing that target accuracy if full knowledge about the unknown solution were given. The theoretical findings are supported and quantified by numerical experiments.

Joint work with Manuela Utzinger (University of Basel).

A5 - December 13, 15:30 – 16:10

A convergent adaptive scheme for hierarchical isogeometric methods

Carlotta Giannelli
INdAM c/o University of Florence, Italy
carlotta.giannelli@unifi.it

Adaptive finite element methods (AFEM) usually rely on an iterative procedure that consists of four fundamental modules. For any iterative step, the adaptive loop starts with the approximation of the solution with respect to the current computational mesh. A posteriori error estimates are then computed in terms of local indicators associated to any single mesh element. Subsequently, a marking strategy selects the elements with higher values of the local error indicator. Finally, a refinement procedure constructs the refined mesh starting from the set of marked elements. In general, the refinement procedure identifies the mesh with an increased level of resolution for the next iteration by refining not only the marked elements, but also a suitable set of elements in their neighbourhood. This may allow to guarantee certain properties of the resulting mesh that preserve the error estimates previously computed. In particular, specific bounds for the number of the non-zero basis functions on any mesh element plays a key role for the development of an adaptivity theory. In order to extend recent results obtained in the AFEM context to the isogeometric setting, we rely on local refinement techniques based on adaptive spline spaces. In particular, by exploiting the truncated basis for hierarchical B-spline spaces together with suitable mesh configurations, we will provide simple residual-type error estimates and prove the convergence of the adaptive procedure.

Joint work with Annalisa Buffa (IMATI "E. Magenes" - CNR, Italy).

A5 - December 13, 17:00 – 17:40

Adaptive methods for nonlinear problems: PDEs and Multiscale Modelling

Charalambos Makridakis
University of Sussex, UK
c.makridakis@sussex.ak.uk

The computation of singular phenomena (shocks, defects, dislocations, interfaces, cracks) arises in many complex systems. For computing such phenomena, it is natural to seek methods that are able to detect them and to devote the necessary computational recourses to their accurate resolution. At the same time, we would like to have mathematical guarantees that our computational methods approximate physically relevant solutions. Our purpose in this talk is to review results and discuss related computational challenges for such nonlinear problems modeled by PDEs. In addition we shall briefly discuss issues related to Micro / Macro adaptive modeling and methods, in particular related to atomistic/continuum coupling in crystalline materials.

A5 - December 13, 17:50 – 18:30

Instance optimality of the maximum strategy

Lars Diening
University of Munich (LMU), Germany
diening@math.lmu.de

We study the adaptive finite element approximation of the Dirichlet problem $-\Delta u = f$ with zero boundary values using newest vertex bisection. Our approach is based on the minimization of the corresponding Dirichlet energy. Our approach works for lower and higher order elements. We show that the maximums strategy attains every energy level with a number of degrees of freedom, which is proportional to the optimal number. As a consequence we achieve instance optimality of the error.

Joint work with Christian Kreuzer (Bochum, Germany) and Rob Stevenson (Amsterdam, Netherlands).

A5 - Poster

A posteriori error estimates for elliptic problems with Dirac measure terms in weighted spaces

Juan Pablo Agnelli
Universidad Nacional de Córdoba and CIEM-CONICET, Argentina
agnelli@famaf.unc.edu.ar

In this work we develop a posteriori error estimates for second order linear elliptic problems with point sources in two- and three-dimensional domains. We prove a global upper bound and a local lower bound for the error measured in a weighted Sobolev space. The weight considered is a (positive) power of the distance to the support of the Dirac delta source term, and belongs to the Muckenhoupt's class A_2. The theory hinges on local approximation properties of either Clément or Scott-Zhang interpolation operators, without need of modifications, and makes use of weighted estimates for fractional integrals and maximal functions. Numerical experiments with an adaptive algorithm yield optimal meshes and very good effectivity indices.

Joint work with Eduardo M. Garau (Universidad Nacional del Litoral and IMAL-CONICET, Argentina) and Pedro Morin (Universidad Nacional del Litoral and IMAL-CONICET, Argentina).

A5 - Poster

MINIMAL SURFACE APPROXIMATION USING ISOGEOMETRIC METHODS

Aníbal Leonardo Chicco Ruiz
UNL - IMAL - CONICET, Argentina
achicco@santafe-conicet.gov.ar

We develop an algorithm to approximate minimal surface problems using isogeometric spaces. In particular we take advantage of the higher regularity of these spaces. The scheme is implemented in the software library IGATOOLS (www.igatools.org), which allows a seamless coding of the so called direct tensor notation. The latter permitting us to implement a Newton method for which we find an explicit formula for the second derivative of the area functional.

Joint work with Sebastián Pauletti (UNL, IMAL, Argentina) and Diego Sklar (UNL, IMAL, Argentina).

A5 - Poster

Quaternion Vortex Methods

Leonardo Traversoni
Uruguay Foreign Ministry, Uruguay
ltraversoni@hotmail.com

QUATERNION VORTEX METHODS

We show the advantages of using the quaternionic expression of some PDEs in this case Navier Stokes in this case to obtain a faster and more simple numerical solution via vortex methods.

The Navier Stokes equations for incompressible viscous flow are:

$$\frac{Du}{Dt} = -\bigtriangledown P + \frac{1}{R} \bigtriangledown^2 u \;\; in\, D \tag{2}$$

$$\bigtriangledown \cdot u = 0 \;\; in\, D \tag{3}$$

$$u = 0 \;\; on\, \partial D \tag{4}$$

Taking ξ as:

$$\xi = \bigtriangledown \times v \tag{5}$$

The vorticity we have the vorticity transport equation:

$$\frac{D\xi}{Dt} = (\xi \cdot \bigtriangledown) u + \frac{1}{R} \bigtriangledown^2 \xi \tag{6}$$

Here u is the velocity, P the pressure, R the Reynolds number.

As $\bigtriangledown \cdot u = 0$ and $\xi = \bigtriangledown \times u$

there exists a vector function $\psi(x)$ such that $u = \bigtriangledown \times \psi$ then

$$\bigtriangledown^2 \psi = -\xi \tag{7}$$

In 3D ψ is the velocity potencial in 2D is the stream function.

Applying ideas of quaternionic and Clifford analysis we can find a transformation into one non-linear equation only for the vorticity ξ. For this reason we use the higher-dimensional version of the Borel-Pompeiu formula:

$$TDu(x) = u(x) - Fu(x) \tag{8}$$

where T is the T-operator (Teodorescu transform), D the Dirac-operator and F the Cauchy integral. The Cauchy integral depends only on the boundary values of u.

That means that if $u = 0$ on the boundary then this part can be deleted of the formula.

Moreover, Du means for a quaternion valued function $(0, u)$ (u is the vector of velocity)

$$Du = (-div\, u,\ rot\, u) \tag{9}$$

As we are working with divergence free vectors and consequently

$$Du = (0,\ rotu)$$

Remembering that

$$rot\ u = \nabla \times u$$

we have that

$u = TDu$ and with $Du = rot\, u = \xi$ it follows

$$u = T\xi \tag{10}$$

This is an expression to describe the velocity u explicitely by the vorticity ξ. If the boundary values of u are not zero but some known quantity then we have this additional known summand F(boundary values of u). The operators T and F are defined as:

$$(T_G u)(x) = -\int_G e(x-y)u(y)dG_y$$

$$(F_\gamma u)(x) = \int_\gamma e(x-y)\alpha(y)u(y)d\gamma_y$$

α is the outer normal to γ at the point y and $e(x)$ the fundamental solution (generalized Cauchy kernel) of the Dirac-operator.

In this way substituting in the above equations we obtain a nonlinear equation in ξ instead a system in u and ξ. To find representation formulas and numerical methods for ξ is one of the goals of the project. Because we have to evaluate only the vorticity (and not in addition the velocity, too) a better efficiency of this approach is expected.

So to apply numerical methods to the equations we only need to have the numerical expressions of the main operators: We show the discrete expression of the main operators used:

$$\Delta u = \sum D^+\_k, jD^-\_k, ju \tag{11}$$

$$D^+D^- = \frac{1}{h}\left[u_{k+1,j} - u_{j,k} - u_{k+1,j+1} + u_{k,j-1}\right] \tag{12}$$

The Teodorescu has also an expression:

$$(T\_hf) = \sum \_y = G\_ke\_h(x-y)f(y)h^3 \tag{13}$$

Where e is the fundamental solution of D We also have that:

$$(F\_hf)(x) = \sum \_y = G\_ke\_h(x-y)\alpha(y)f(y)h^2 \tag{14}$$

So now we only have to apply the above in the classical vortex method as follows: 1) Create an initial particle field that approximates ξ_i , by placing uniformly spaced particles into the support of ξ_i and by setting their vectorial circulation to the local value of ξ_i

2) Create an triangulation $S = S_i$ of the boundaries and place immovable vortex particles on the triangles' centres. 3) Use a standard time-stepping technique, e.g., a Runge-Kutta method or a multistep method,

for advancing the ODEs in time. In order to evaluate the vorticities and their gradients in each (sub-)step, do the following: ξ_i at each particle location as well as at the quadrature points on the boundaries with the help of the Biot-Savart law b) Compute

$\bigtriangledown\xi_i$

at each particle location, again by using the Biot-Savart law. c) Compute the unknown vortex sheet strength on the boundaries. d) Now pretend the particles on the boundaries are ordinary particles. Use the Biot- Savart law (6) in order to compute ξ_i

and $\bigtriangledown\xi_i$

at each particle inside the domain. 4. Remove any particles that might have escaped the fluid domain. This should only rarely happen if the surface triangulation is sufficiently refined. 5. Repeat steps 3 and 4 until the requested termination time is reached

---

Workshop A6
Real Number Complexity

Organizers: Saugata Basu – Carlos Beltrán – Felipe Cucker

A6 - December 11, 14:30 – 15:00

Some Results on the Complexity of the Eigenvalue Problem

Diego Armentano
Universidad de La República, Uruguay
diegoax@gmail.com

In this talk we present an algorithm for the eigenvalue(eigenvector) problem which runs on average polynomial time over the space of n by n complex matrices with Gaussian entries.

Joint work with Carlos Beltrán (Universidad de Cantabria, Spain), Peter Bürgisser (Technische Universität Berlin, Germany), Felipe Cucker (City University of Hong Kong, Hong Kong) and Michael Shub (City University of New York, USA).

A6 - December 11, 15:00 – 15:30

On sparse polynomial solving

Gregorio Malajovich
Universidade Federal do Rio de Janeiro, Brasil
gregorio.malajovich@gmail.com

Most of the rigorous theory of path-following algorithms for polynomial system solving assumes dense or dense multi-homogeneous polynomial systems. However, typical examples arising from applications tend to have a sparse structure which calls for theorems.

I will report on recent advances on sparse polynomial solving by homotopy, including mixed volume computation and path-following on toric varieties.

A6 - December 11, 15:35 – 16:25

Variational analysis in the light of semi-algebraic geometry

Aris Daniilidis
Universidad de Chile, Chile
arisd@dim.uchile.cl

In this talk we survey results of the so-called Semi-algebraic (Tame) Variational Analysis, with emphasis to the desingularization of (variational) criticality and its consequences to the asymptotic analysis of (nonsmooth) dynamical systems.

Joint work with The talk is based on several works obtained in collaboration with J. Bolte (Université Toulouse 1, France), D. Drusvyatskiy (University of Washington, USA) and A. S. Lewis (Cornell University, USA).

A6 - December 11, 17:30 – 18:00

Elementary recursive degree bounds for Positivstellensatz, Hilbert 17th problem and Real Nullstellensatz (part I)

Daniel Perrucci
Universidad de Buenos Aires, Argentina
perrucci@dm.uba.ar

Hilbert 17th problem is to express a non-negative polynomial as a sum of squares of rational functions. The original proof by Artin is non-constructive and gives no information on the degree bounds.

A more general problem is to give an identity which certifies the unrealizability of a system of polynomial equations and inequalities. The existence of such an identity is guaranteed by the Positivstellensatz. Hilbert 17th problem, as well as Real Nullstellensatz follow easily from such identities.

In these two talks, we explain our new constructive proof which provides elementary recursive bounds for the Positivstellensatz and Hilbert 17 problem, namely a tower of five levels of exponentials.

Joint work with Henri Lombardi (Université de Franche-Comté, France) and Marie-Françoise Roy (Université de Rennes 1, France).

A6 - December 11, 18:00 – 18:30

Elementary recursive degree bounds for Positivstellensatz, Hilbert 17th problem and Real Nullstellensatz (part II)

Marie-Françoise Roy
Université de Rennes 1, France
marie-francoise.roy@univ-rennes1.fr

Hilbert 17th problem is to express a non-negative polynomial as a sum of squares of rational functions. The original proof by Artin is non-constructive and gives no information on the degree bounds.

A more general problem is to give an identity which certifies the unrealizability of a system of polynomial equations and inequalities. The existence of such an identity is guaranteed by the Positivstellensatz. Hilbert 17th problem, as well as Real Nullstellensatz follow easily from such identities.

In these two talks, we explain our new constructive proof which provides elementary recursive bounds for the Positivstellensatz and Hilbert 17 problem, namely a tower of five levels of exponentials.

Joint work with Henri Lombardi (Universite de Franche-Comté, France) and Daniel Perrucci (Universidad de Buenos Aires, Argentina).

A6 - December 11, 18:30 – 19:00

Towards a Broader View of Theory of Computing – Part 1

Narendra Karmarkar
Indian Institute of Technology, Bombay, India
narendrakarmarkar@yahoo.com

Beginning with the projectively invariant method for linear programming, interior point methods have led to powerful algorithms for many difficult computing problems, in combinatorial optimization, logic, number theory and non-convex optimization. Algorithms for convex optimization benefitted from many pre-established ideas from classical mathematics, but non-convex problems require new concepts. This three part series outlines some of these concepts– computational models based on the concept of the continuum, algorithms invariant w.r.t. projective , bi-rational , and bi-holomorphic transformations on co-ordinate representation, extended proof systems for more efficient certificates of optimality, extensions of Grassmann's extension theory, efficient evaluation methods for the effect of exponential number of constraints , theory of connected sets based on graded connectivity, theory of curved spaces adapted to the problem data, and concept of " relatively" algebraic sets in curved space.

Models of Computation provide mathematical abstractions of basic data objects and operations on them available as building blocks. This effectively decouples design of algorithms for complex tasks from lower level details of how the underlying hardware implements the building blocks. The Turing machine model uses strings of 0's and 1's and finite state machines. Careful study of the work of early pioneers – Turing,Von Neumann and Godel – shows that they were acutely aware of the limitations of this model for comprehensive understanding of the fundamental aspects of computation. BSS model uses real or complex numbers as data objects and algebraic operations (including comparisons in real case). This is more natural for many algorithms in numerical analysis. Various computing models can be organized in a similar way as Cantor had organized infinite sets—by cardinal number of the set of all possible machines and data objects in the model. Staying within the same cardinal number, a more powerful approach is to use further extension, e.g real analytic functions or algebraic closure of meromorphic functions over suitable domains. Operations include algebraic as well as analytic operations. i.e. integration and differentiation are regarded as binary operations. (specification of the contour of integration is one of the input operands) All such models are collectively referred to as continuum computing. Time permitting, more topics from the list above will be covered in the first part.

A6 - December 12, 14:30 – 15:00

De Rham Cohomology and Ordinary Differential Equations

Peter Scheiblechner
Lucerne University of Applied Sciences and Arts, Switzerland
peter.scheiblechner@hslu.ch

We describe an EXPSPACE-algorithm for computing the de Rham cohomology of the complement of an affine hypersurface over the complex numbers. The best previously known algorithm for this problem (cylindrical algebraic decomposition) needs double exponential time. Our algorithm follows the lines of a proof of Monsky for the finite dimensionality of the de Rham cohomology. It reduces the computation of the cohomology to certain systems of ordinary differential equations with Laurent polynomial coefficients. We study the complexity of the solutions of such systems.

A6 - December 12, 15:00 – 15:30

Can everything be computed? - On the Solvability Complexity Index and Towers of Algorithms

Anders Hansen
University of Cambridge, United Kingdom
ach70@cam.ac.uk

This talk addresses some of the fundamental barriers in the theory of computations. Many computational problems can be solved as follows: a sequence of approximations is created by an algorithm, and the solution to the problem is the limit of this sequence (think about computing eigenvalues of a matrix for example). However, as we demonstrate, for several basic problems in computations (computing spectra of infinite dimensional operators, solutions to linear equations or roots of polynomials using rational maps) such a procedure based on one limit is impossible. Yet, one can compute solutions to these problems, but only by using several limits. This may come as a surprise, however, this touches onto the boundaries of computational mathematics. To analyze this phenomenon we use the Solvability Complexity Index (SCI). The SCI is the smallest number of limits needed in order to compute a desired quantity. In several cases (spectral problems, inverse problems) we provide sharp results on the SCI, thus we establish the absolute barriers for what can be achieved computationally. For example, we show that the SCI of spectra of self-adjoint infinite matrices is equal to two, thus providing the first algorithms to compute such spectra in two limits. Moreover, we establish barriers on error control. We prove that no algorithm can provide error control on the computational spectral problem or solutions to infinite-dimensional linear systems. In particular, one can get arbitrarily close to the solution, but never knowing when one is "epsilon" away. In addition, we provide bounds for the SCI of spectra of classes of Schrodinger operators, thus we affirmatively answer the long standing question on whether or not these spectra can actually be computed. Finally, we show how the SCI provides a natural framework for understanding barriers in computations. In particular, we demonstrate how the impossibility result of McMullen on polynomial root finding with rational maps in one limit, and the framework of Doyle and McMullen on solving the quintic in several limits, can be put in the SCI framework.

Joint work with Jonathan Ben-Artzi (University of Cambridge), Olavi Nevanlinna (Aalto University), Markus Seidel (TU Chemnitz).

A6 - December 12, 15:35 – 16:25

Probabilistically Checkable Proofs over the Reals

Klaus Meer
Brandenburg Technical University Cottbus - Senftenberg, Germany
meer@informatik.tu-cottbus.de

One fruitful source of interesting problems in structural complexity theory over the reals since its introduction by Blum, Shub, and Smale in 1989 has been the investigation of major theorems known in classical (Turing) complexity within the real number framework. Prominent examples are the P versus NP question, decidability algorithms for problems in NP over the reals, and the theorems by Ladner and Toda, to mention a few only.

A cornerstone in Turing complexity theory is the PCP theorem shown in the early 1990's by Arora et al. It gives an alternative characterization of the complexity class NP via so-called probabilistically checkable proofs. Roughly speaking, it shows that a verification algorithm for instances of problems in NP can be designed such that a false verification proof can be detected with high probability by reading constantly many proof bits only. The theorem has shown huge impact on approximation problems as well.

It is thus natural to ask whether a similar characterization holds for the class $NP_{\mathbb{R}}$ of problems in NP over the reals. This boils down to the question whether there is a (randomized) procedure that verifies a solvability proof of a polynomial system over the reals by only inspecting a constant number of real components in the proof. Obviously, the natural $NP_{\mathbb{R}}$-verification proof that takes a zero and just plugs it into the system fails.

In the talk we shall report on techniques and results that we have been working on during the last couple of years in order to design real number PCPs. We explain the role of property testing and shall outline the proof of the corresponding PCP theorem both in the real and complex number BSS model. A focus will be on highlighting the differences (and non-differences) to the existing proofs of the classical PCP theorem.

Joint work with Martijn Baartse (Brandenburg Technical University Cottbus - Senftenberg, Germany).

A6 - December 12, 17:00 – 17:30

Geometric Complexity Theory, Tensor Rank, and Representation Theory

Christian Ikenmeyer
Texas A&M University, USA
ciken@math.tamu.edu

Mulmuley and Sohoni conjectured that the permanent vs determinant problem can be resolved using explicit so-called representation theoretic occurence obstructions. It is still unknown whether these objects exist or not, even for small examples. We show that in the simpler (but still highly interesting) setting of matrix multiplication these obstructions indeed exist! We prove the lower bound $R(M_m) \geq \frac{3}{2}m^2 - 2$ on the border rank of m x m matrix multiplication by exhibiting explicit representation theoretic occurence obstructions. While this bound is weaker than the one recently obtained by Landsberg and Ottaviani, these are the first significant lower bounds obtained within the GCT program. Behind the proof is basically Schur-Weyl duality and an explicit description of highest weight vectors in terms of combinatorial objects.

Joint work with Peter Bürgisser (Technische Universität Berlin, Germany).

A6 - December 12, 17:30 – 18:00

On the intersection of a sparse curve and a low-degree curve: A polynomial version of the lost theorem

Pascal Koiran
Ecole Normale Supérieure de Lyon, France
pascal.koiran@ens-lyon.fr

Consider a system of two polynomial equations in two variables:

$$F(X,Y) = G(X,Y) = 0$$

where $F \in \mathbb{R}[X,Y]$ has degree $d \geq 1$ and $G \in \mathbb{R}[X,Y]$ has t monomials. We show that the system has only $O(d^3t + d^2t^3)$ real solutions when it has a finite number of real solutions. This is the first polynomial bound for this problem. In particular, the bounds coming from the theory of fewnomials are exponential in t, and count only nondegenerate solutions. More generally, we show that if the set of solutions is infinite, it still has at most $O(d^3t + d^2t^3)$ connected components.

By contrast, the following question seems to be open: if F and G have at most t monomials, is the number of (nondegenerate) solutions polynomial in t?

The authors' interest for these problems was sparked by connections between lower bounds in algebraic complexity theory and upper bounds on the number of real roots of "sparse like" polynomials.

Joint work with Natacha Portier and Sébastien Tavenas.

A6 - December 12, 18:00 – 18:30

A polynomial homotopy random walk

Anton Leykin
Georgia Tech, USA
leykin@math.gatech.edu

Given a one parameter family of polynomial systems with complex coefficients, we develop a new way of tracking an initial system-solution pair to a solution of the target system. While the theoretical complexity is believed to be, at best, the same as for traditional approaches, there are practical advantages to this method in certain scenarios. (Work in progress.)

A6 - December 13, 14:30 – 15:00

Universal components of random algebraic sets

Damien Gayet
Institut Fourier, Grenoble 1, France
gayetd@ujf-grenoble.fr

I will explain that any compact hypersurface S of R^n appears with a positive probability c_S as a component of a random algebraic hypersurface of high degree d, with c_S not depending on d.

Joint work with Jean-Yves Welschinger (Institut Camille Jordan, Lyon 1, France).

A6 - December 13, 15:00 – 15:30

On the number of zeros of E-polynomials

Gabriela Jeronimo
Universidad de Buenos Aires - CONICET, Argentina
jeronimo@dm.uba.ar

An E-polynomial in a single variable with integer coefficients is a real function of the form $P(x, e^{h(x)})$, where $P(X,Y) \in \mathbb{Z}[X,Y]$ and $h \in \mathbb{Z}[X]$. These functions form a particular subclass of the so-called Pfaffian functions introduced by Khovanskii in the 70's. As proved by Vorobjov in 1992, the consistency problem for (multivariate) E-polynomials is algorithmically decidable.

We will present an algorithm for the computation of the number of zeros of an E-polynomial in $\mathbb{R}$ and in an interval of $\mathbb{R}$. Our algorithm is inspired in the well-known Sturm's Theorem for real univariate polynomials. It relies on the construction, from the polynomials P and h, of adequate sequences of E-polynomials and the sign determination of their evaluation at algebraic real numbers. We prove that the number of real zeros of $P(x, e^{h(x)})$ can be computed from the number of sign changes of these sequences evaluated at finitely many points that are obtained throughout the construction. In order to determine sign changes, we also develop an algorithmic procedure to compute the sign of an E-polynomial evaluated at an algebraic real number given by its Thom encoding.

Joint work with María Laura Barbagallo (Universidad de Buenos Aires - CONICET), Juan Sabia (Universidad de Buenos Aires - CONICET).

A6 - December 13, 15:30 – 16:00

An excursus into Zeta Mahler measure functions

Luis M. Pardo
University of Cantabria, Spain
luis.m.pardo@gmail.com

In 2009, H. Akatsuka introduced the Zeta Mahler measure function for polynomials and Laurent polynomials. This talk re-considers the main ideas in his introduction and adapts and generalizes his formulation in order to deal with other integrable quantities. We firstly exhibit some preliminary applications: we may determine, for instance, the domain of holomorphy of the Zeta Mahler functions of linear and non-linear condition numbers or random homogeneous polynomials, for instance. The focus of our interest in this notion is not dealing with these immediate applications, but the bottleneck in the study of the arithmetic height of the discriminant variety. Finally, combining this technique and other estimates, we may exhibit a beautiful and exact description of the arithmetic height of the discriminant variety $\Sigma_{(d)}$, in the case of complete intersection zero-dimensional projective varieties. This talk is based in a joint work with Mario Pardo, from the University of Cantabria.

Joint work with Mario Pardo (Universidad de Cantabria, Spain).

A6 - December 13, 16:00 – 16:30

Counting the number of components of random real hypersurfaces

Antonio Lerario
Institut Camille Jordan, Lyon, France
anto.lerario@gmail.com

To determine the average number of real zeroes of a univariate polynomial whose coefficients are random variables is a classical and well studied problem. A natural way to generalize it is to ask for the average number of connected components of the zero set of a random polynomial in several variables. This approach is much influenced by a random approach to Hilbert's Sixteenth Problem, to study the number and the arrangement in the projective space of the components of a real algebraic hypersurface.

The answer to the above question (both in the univariate and the multivariable case) strongly depends on the choice of the probability distribution.

In this talk I will show that, if the probability distribution is Gaussian and has no preferred points or directions in the projective space, the case of several variables can be reduced to the classical univariate problem. More precisely, the number of connected components of a random hypersurface in $\mathbb{R}P^n$ of degree d has the same order of the number of points of intersection of this hypersurface with a fixed projective line, raised to the n-th power.

Joint work with Yan V. Fyodorov (School of Mathematical Sciences, Queen Mary University of London) and Erik Lundberg (Department of Mathematics, Florida Atlantic University).

A6 - December 13, 17:00 – 17:30

On the computation of roadmaps of real algebraic sets

Eric Schost
Western University, Canada
eschost@uwo.ca

We consider the question of constructing roadmaps of real algebraic sets, which was introduced by Canny to answer connectivity questions and solve motion planning problems. Canny's algorithm is recursive, but decreases the dimension only by one through each recursive call.

In this talk, we present a divide-and-conquer version of Canny's algorithm, where the dimension is halved through each recursive call. The input is a compact complete intersection V given by polynomials of degree D in n variables; we also have to assume that V has a finite number of singular points.

We also mention results of a preliminary implementation, and may indicate how the calculation of points on some Thom-Boardman strata could explain some of these results.

Joint work with Mohab Safey El Din (Universite Pierre et Marie Curie (Paris 6), France).

A6 - December 13, 17:30 – 18:00

The Betti numbers of an intersection of random quadrics

Erik Lundberg
Florida Atlantic University, USA
elundber@fau.edu

Let X be an intersection of k quadrics chosen at random from the so-called Kostlan ensemble. As the number of variables increases, we study the asymptotics of each Betti number of X and show that the expected ith Betti number is asymptotically one. In particular, an intersection of quadrics is asymptotically connected on average. In the case of an intersection of $k = 2$ quadrics, we give additional detail on the sum of all Betti numbers, providing an asymptotic with two orders of precision. The proofs apply the Agrachev-Lerario spectral sequence from Algebraic Topology combined with results from Random Matrix Theory. The case of three quadrics leads to considering a new model for random curves based on determinantal representations.

Joint work with Antonio Lerario (Lyon 1, France).

Workshop B1
Approximation Theory

Organizers: Nira Dyn – Tom Lyche – Holger Wendland

B1 - December 15, 14:35 – 15:25

Greedy approximation of a solution manifold

Wolfgang Dahmen
RWTH Aachen, Germany
dahmen@igpm.rwth-aachen.de

Many design or optimization tasks in scientific computation require a frequent(even online) evaluation of solutions to parameter dependent families of partial differential equations describing the underlying model. This is often only feasible when the model is suitably reduced. The so called Reduced Basis Method is a model reduction paradigm that has recently been attracting considerable attention since it aims at certifying the quality of the reduced model through a-posteriori error bounds. The central idea is to approximate the solution manifold, comprised of all solutions obtained when the parameters range over the given parameter domain by the linear hull of possibly few snapshots from that manifold so as to still guarantee that the maximal error stays below a given target tolerance. Each parameter query, e.g. in an optimization process, requires then only solving a relatively small problem in the reduced space. This can be viewed as finding a problem dependent dictionary with respect to which all solutions are nearly sparse. This talk highlights the basic ideas behind this method. In particular, it is indicated under which circumstances certain greedy strategies for finding the generating snapshots are optimal in the sense that the corresponding subspaces realize the rate of the Kolmogorov n-widths even for problem classes not covered by conventional strategies.

B1 - December 15, 15:30 – 15:55

Approximation of freeform surfaces with polyhedral patterns

Helmut Pottmann
KAUST, Saudi Arabia, and TU Wien, Austria
pottmann@geometrie.tuwien.ac.at

Polyhedral meshes, i.e. meshes with planar faces, recently received a lot of interest because of their potential applications in architecture and industrial design. Avoiding triangle meshes because of their high node complexity, research concentrated mainly on polyhedral quad meshes. They possess an elegant treatment within discrete differential geometry and are capable of approximating arbitrary shapes. However, they are strongly linked to the curvature behavior of the surfaces to be approximated and may not possess sufficient flexibility to satisfy the design intent. As an alternative, various types of patterns different from the quad grid have been investigated, both in real projects and in geometric computing. We report on our recent progress on polyhedral meshes which are combinatorially equivalent to well-known 2D patterns, analyze their flexibility in approximating freeform shapes, discuss ways to handle the arising new forms of smoothness and suggest a computational framework suitable for interactive design and approximation.

Joint work with Caigui Jiang, Jun Wang, Chengcheng Tang, Peter Wonka and Johannes Wallner.

B1 - December 15, 16:00 – 16:25

Convolution operations in curve and surface modeling

Thomas Grandine
Boeing Computer Services, USA
thomas.a.grandine@boeing.com

Motivated by the need to develop empirical models of cured composite laminated parts, convolution-based techniques of smoothing curve and surface models have recently been investigated. These convolution operations for tensor product splines have been found to be very effective, not only for building the intended models, but also for modeling tooling and machining. The technique so far appears to be very promising, and it appears to offer the potential to solve some of the data complexity issues that arise in the practical modeling of real world manufactured parts. This talk will describe the investigations that have so far been performed and offer ideas for further development

---

B1 - December 15, 17:05 – 17:55

Linear Differential Operators on Spline Spaces and Spline Vector Fields

Tatyana Sorokina
Towson University, USA
tsorokina@towson.edu

We consider the application of standard differentiation operators to polynomial spline spaces and spline vector fields defined on triangulations. In particular, we explore the use of Bernstein Bézier techniques for answering questions such as: what are the images or the kernels, and their dimensions, of partial derivative, gradient, divergence, curl, or Laplace, operators. We also describe applications of our results to finite element methods.

Joint work with Peter Alfeld (University of Utah).

B1 - December 15, 18:00 – 18:25

Algebraic tools for the study of spline spaces

Nelly Villamizar
RICAM, Austria
nvillami@gmail.com

A spline space attached to a partitioned domain is the vector space of functions which are polynomials (up to certain degree) on each piece of the partition and have a fixed order of global smoothness. In this talk, we will address the problem of finding the dimension of the spline space associated to a triangulation or a tetrahedral partition of a region in the plane or in the three dimensional space, respectively. Applying homological techniques and exploring connections of splines with ideals generated by powers of linear forms, we establish formulas for lower and upper bounds on the dimension of the spline space for any given degree and order of smoothness. The homological construction gives an insight into ways of computing the exact dimension for a given partition, and brings to light connections between the theory of splines and classical problems in algebraic geometry.

B1 - December 16, 14:35 – 15:25

Wavelet decompositions of Random Forests

Shai Dekel
GE Global Research & Tel-Aviv University, Israel
Shai.Dekel@GE.com

In the talk we will review how Approximation Theory can be applied to solve some of the challenges of Machine Learning. Tools such as the tree-based Random Forest and the Gradient Boosting Machine are popular and powerful machine learning algorithms that are also employed as part of 'Deep Learning' systems. Constructing the right form of wavelet decomposition of these tools allows establishing ordering of their decision nodes: from 'significant' features to 'less significant' to 'insignificant' noise. Consequently, simple wavelet techniques can be used to overcome the presence of noise and misclassifications in the training sets and compress large scale neural networks.

B1 - December 16, 15:30 – 15:55

Tree Algorithms for Classification

Peter Binev
University of South Carolina , USA
binev@math.sc.edu

The solution of the problem of binary classification is often presented as a set of points at which one predicts a positive outcome assuming that a negative outcome is expected at the points of the complement set. Given a family of sets the goal is to find an element for which the risk of misclassification is as small as possible. We want to find an effective algorithm that based on a collection of query points identifies with high probability a good approximation in terms of minimizing the risk. As usual, no prior knowledge is assumed about the underlying probability measure related to the classification. We consider families of sets defined via tree structures and propose a convergence analysis based on nonlinear approximation theory, which allows us to significantly weaken the usual assumptions that are made to establish a given convergence rate. Algorithms using occupancy trees and decorated trees are presented to show one possible efficient realization of the general theory.

B1 - December 16, 16:00 – 16:25

Using Semidefinite Programming in Approximation Theory

Simon Foucart
University of Georgia, USA
foucart@math.uga.edu

This talk reports on some ongoing work that uncovers many semidefinite relaxations to classic Approximation Theory problems. These include best approximations in the max-norm by trigonometric polynomials, algebraic polynomials, rational functions, and splines. One may deal with unconstrained, one-sided, monotone, or simultaneous approximations alike. Solving the associated semidefinite programs numerically gives new insight on various results and conjectures.

B1 - December 16, 17:05 – 17:55

Weighted D-T moduli revisited and applied

Dany Leviatan
Tel Aviv University, Israel
leviatan@math.tau.ac.il

We introduce weighted moduli of smoothness for functions $f \in L_p[-1,1] \cap C^{r-1}(-1,1)$ $r \geq 1$, that have an $(r-1)$st absolutely continuous derivative in $(-1,1)$ and such that $\varphi^r f^r \in L_p[-1,1]$, where $\varphi(x) = (1-x^2)^{1/2}$. These moduli are equivalent to certain weighted D-T moduli, but our definition is more transparent and simpler. In addition, instead of applying these weighted moduli to weighted approximation, which was the purpose of the original D-T moduli, we apply these moduli to obtain Jackson-type estimates on the approximation of functions in $L_p[-1,1]$ (no weight), by means of algebraic polynomials. We also have inverse theorems that yield characterization of the behavior of the derivatives of the function by means of its degree of approximation.

Joint work with K. Kopotun (University of Manitoba, Canada) and I. A. Shevchuk (Ukrainian National Academy of Sciences, Kiev).

B1 - December 16, 18:00 – 18:25

Estimating the n-width of solution manifolds of parametric PDE's

Albert Cohen
Université Paris VI, France
cohen@ann.jussieu.fr

Numerous computational model reduction methods, such as reduced basis and its generalization, proper orthogonal decomposition, generalized empirical interpolation, rely on the implicit assumption that the solution manifold, that gathers the solution to a PDE as parameters vary, can be well approximated by low dimensional spaces. This is made more precise by assuming that the Kolmogorov n-width of the manifold has certain decay. In this talk, I shall discuss strategies that allow to rigorously establish rates of decay for the Kolmogorov n-width of solution manifolds associated with relevant parametric PDE's, where the parameters are infinite dimensional. One key result is that the rate of decay of n-width of sets in Banach space is almost preserved under the action of holomorphic maps.

Joint work with Ronald DeVore (Texas A&M University).

B1 - December 17, 14:35 – 15:25

Stable reconstruction from Fourier samples

Alexei Shadrin
University of Cambridge, UK
A.Shadrin@dampt.cam.ac.uk

For an analytic and periodic function f, the m-th partial sums of its Fourier series converge exponentially fast in m, but such rapid convergence is destroyed once periodicity is no longer present (because of the Gibbs phenomenon at the end-points).

We can restore higher-order convergence, e.g., by reprojecting the slowly convergent Fourier series onto a suitable basis of orthogonal algebraic polynomials, however, all exponentially convergent methods appear to suffer from some sort of ill-conditioning, whereas methods that recover f in a stable manner have a much slower approximation rate.

We give to these observations a definite explanation in terms of the following fundamental stability barrier: the best possible convergence rate for a stable reconstruction from the first m Fourier coefficients is root-exponential in m.

B1 - December 17, 15:35 – 16:25

α-Molecules: Wavelets, Shearlets, and beyond

Gitta Kutyniok
Technische Universität Berlin), Germany
kutyniok@math.tu-berlin.de

The area of applied harmonic analysis provides a variety of multiscale systems such as wavelets, curvelets, shearlets, or ridgelets. A distinct property of each of those systems is the fact that it sparsely approximates a particular class of functions. Some of these systems even share similar approximation properties such as curvelets and shearlets which both optimally sparsely approximate functions governed by curvilinear features, a fact that is usually proven on a case-by-case basis for each different construction. The recently developed framework of parabolic molecules, which includes all known anisotropic frame constructions based on parabolic scaling, provides a unified concept for a sparse approximation results of such systems.

In this talk we will introduce the novel concept of α-molecules which allows for a unified framework encompassing most multiscale systems from the area of applied harmonic analysis with the parameter α serving as a measure for the degree of anisotropy. The main result essentially states that the cross-Gramian of two systems with the same degree of anisotropy exhibits a strong off-diagonal decay. One main consequence we will discuss is that all such systems then share similar approximation properties, and desirable approximation properties of one can be deduced for virtually any other system with the same degree of anisotropy.

Joint work with P. Grohs (ETH Zurich), S. Keiper (TU Berlin) and M. Schäfer (TU Berlin).

B1 - December 17, 17:05 – 17:55

Recent progress on boundary effects in kernel approximation

Thomas Hangelbroek
University of Hawai, USA
hangelbr@math.hawaii.edu

The existence of a boundary in kernel approximation is known to drive down rates of convergence in many cases, using kernels with centers restricted to a bounded domain causes approximation orders to be saturated at a low rate (roughly half the rate of the corresponding boundary-free rates). Another unpleasant eect is that although alternative bases like Lagrange and local Lagrange functions are known to be stable and well-localized in boundary-free kernel approximation, the proof of this fact does not generalize to re- gions with boundary. Indeed, there is strong evidence that in the presence of a boundary, Lagrange functions decay at a rate that is too slow to be useful. In this talk we present recent advances in kernel approximation that treat both of the aforementioned boundary eects. We begin by discussing approximation results that over- come the low saturation order imposed by the boundary. This is an expansion from the previously understood case of surface splines on Euclidean regions to more general kernels acting on

Riemannian manifolds. We follow this by presenting local basis constructions for bounded regions which yield Lp-stability results and Bernstein inequalities.

B1 - December 17, 18:00 – 18:25

Series kernels for high dimensional reconstruction problems

Christian Rieger
University of Bonn, Germany
rieger@ins.uni-bonn.de

In this talk, we will discuss series kernels and their applications in high dimensional problems. In many problems we are able to determine an application-adapted reproducing kernel which is however typically not available in closed form. This makes it necessary to approximate the kernel itself. In most cases, one can derive an infinite series expansion of the kernel. For numerical purposes, however, this infinite series has to be truncated. We will present some approximation properties of kernel methods based on such truncated kernels. Further, we present how such series expansion can be obtained from the application. As a model problem, we will focus on parametric partial differential equations as they appear in the field of uncertainty quantification. As the resulting kernels are in many cases smooth, we are able to show exponential convergence rates in the parameter discretization. Furthermore we use sampling inequalities to analyze non-intrusive methods. In particular, we derive error estimates which contain the error of solving the partial differential equation for a fixed parameter. This allows us to couple these two errors and to derive a final convergence rate.

Workshop B2
Computational Topology and Geometry

Organizers: Herbert Edelsbrunner – Joel Hass – Alex Nabutovsky

---

B2 - December 15, 14:35 – 15:25

Measuring the geometric similarities of genus-zero surfaces

Patrice Koehl
University of California, Davis, USA
pakoehl@ucdavis.edu

Finding efficient algorithms to describe, measure and compare shapes is a central problem in numerous disciplines that generate extensive quantitative and visual information. Among these, biology occupies a central place. Registration of brain anatomy for example is essential to many studies in neurobiology; at a molecular level, comparison of protein shapes is a key step in understanding the relationships between their functions. In this talk I will introduce the idea of a globally optimal conformal mapping between two (discrete) surfaces of genus zero as one method to solve this problem. In this approach, the whole mesh representing the source surface is warped onto the target surface, using the mapping defined through the composition of discrete conformal mappings of the surfaces onto the sphere and the Möbius transformation between these mappings. The Möbius transformation is then optimized to lead to minimal distortion between the source mesh and its image, where distortion is measured as difference from isometry. I will show that this approach leads to the definition of a metric in the space of genus-zero surfaces. I will describe the implementation of this approach and its applications on biological examples, from brain surface matching to testing how round proteins are.

Joint work with Joel Hass (University of California, Davis, USA).

B2 - December 15, 15:30 – 15:55

Induced Matchings of Barcodes and the Algebraic Stability of Persistence

Ulrich Bauer
TU München, Germany
mail@ulrich-bauer.org

We define a simple, explicit map sending a morphism $f : M \to N$ of pointwise finite dimensional persistence modules to a matching between the barcodes of M and N. Our main result is that, in a precise sense, the quality of this matching is tightly controlled by the lengths of the longest intervals in the barcodes of $\ker f$ and $\operatorname{coker} f$. As an immediate corollary, we obtain a new proof of the algebraic stability of persistence, a fundamental result in the theory of persistent homology. In contrast to previous proofs, ours shows explicitly how a δ-interleaving morphism between two persistence modules induces a δ-matching between the barcodes of the two modules. Our main result also specializes to a structure theorem for submodules and quotients of persistence modules.

Joint work with Michael Lesnick (IMA, Minneapolis, MN).

B2 - December 15, 17:00 – 17:25

Configuration spaces of hard disks in an infinite strip

Matthew Kahle
Ohio State University, United States
mkahle@math.osu.edu

We study the configuration space $C(n, w)$ of n non-overlapping unit-diameter disks in an infinite strip of width w. We present an asymptotic formula for the kth Betti number of $C(n, w)$, for fixed k and w as $n \to \infty$. We find that there is a striking phase transition: for $w > k$ the kth homology is stable and is isomorphic to the kth homology of the configuration space of points. But for $w \leq k$, the kth homology is wildly complicated, growing exponentially fast with n.

Joint work with Robert MacPherson (Institute for Advanced Study).

B2 - December 15, 17:30 – 17:55

Random 3-manifolds.

Joseph Maher
College of Staten Island, CUNY, USA
joseph.maher@csi.cuny.edu

We discuss some recent results on random 3-manifolds, including recent work on the Casson invariant of random Heegaard splittings.

Joint work with Alexander Lubotzky (Hebrew University) and Conan Wu (Princeton University).

B2 - December 15, 18:00 – 18:25

Variations on Topological Complexity

Hellen Colman
Wright College, Chicago, USA
hcolman@ccc.edu

We give a succinct introduction to Farber's topological complexity. This number is a homotopy invariant which reflects the complexity of the problem of constructing a motion planning algorithm in the configuration space of a mechanical system. We introduce a groupoid invariant carrying an interpretation in terms of the motion planning problem for a robot when its configuration space exhibits symmetries. This number is an interesting invariant in itself to measure the complexity of motion planning algorithms in situations that might be modeled by a group action. In particular it could provide a model for the planning of a robot transporting itself by means of two different types of motions. For example, a model for a robot traveling in the physical space by guided tracks and/or air transportation. This is joint work with Andres Angel.

Joint work with Andres Angel (Universidad de los Andes, Colombia).

B2 - December 16, 14:30 – 14:55

Distributed Computation of Persistent Homology using the Blowup Complex

Dmitriy Morozov
Lawrence Berkeley National Laboratory, USA
dmitriy@mrzv.org

This talk will describe an algorithm to distribute computation of persistent homology over multiple processors, by subdividing the domain into regions and combining the results using the Mayer-Vietoris blowup complex.

Joint work with Ryan Lewis (Stanford University) and Gunnar Carlsson (Stanford University).

B2 - December 16, 15:00 – 15:25

Algebraic Morse-Forman-Conley theory

Marian Mrozek
Jagiellonian University, Poland
mrozek@ii.uj.edu.pl

In late 90' R. Forman introduced the concept of a combinatorial vector field on a CW complex and presented a version of Morse theory for acyclic combinatorial vector fields. He also studied combinatorial vector fields without acyclicity assumption, introduced the concept of a chain recurrent set and proved Morse inequalities in this setting.

In years 2005-06 the discrete Morse theory of Forman has been generalized by several authors from the case of CW complexes to the purely algebraic case of chain complexes with a distinguished basis.

In this talk we present the Morse-Conley theory in such a purely algebraic setting. This, in particular, generalizes the definition of an isolated invariant set and its Conley index proposed recently by T. Kaczynski, M. Mrozek and Th. Wanner for simplicial complexes. Moreover, we define attractors, repellers, attractor repeller-pairs, and Morse decompositions and extend to this combinatorial/algebraic setting some classical results of Morse-Conley theory for flows.

B2 - December 16, 15:30 – 15:55

Persistent Objects

Amit Patel
Institute for Advanced Study, USA
amit@ias.edu

For a continuous map $f : X \to R$ to the reals, there is a persistent homology group for each interval $[r, s]$. If X_r is the r-sublevel set of f and X_s the s-sublevel set of f, then the persistent homology group is image of the homomorphism $H_d(X_r) \to H_d(X_s)$ induced by the inclusion $X_r \subset X_s$. This is the homology that spreads out over the interval $[r, s]$. Recently, it has been shown that the persistent homology group can be defined for any map $f : X \to M$, where M is an oriented manifold. If U is an open set of M, then the persistent homology over U is a subgroup of $H_d(f^{-1}(U))$ that spreads out over U. The notion of persistent homology categorifies. Let $F : D \to C$ be a diagram in a category C. Under some mild assumptions on C, there is a notion of a persistent object for F. This is the object in C that spreads out over then entire diagram. In this talk, I will present the persistent homology group of maps to the reals, the persistent homology group for maps to any oriented manifold, and the persistent object.

B2 - December 16, 16:00 – 16:25

Beyond Convexity: New Perspectives in Computational Optimization

Narenda Karmarkar
, India
narendrakarmarkar@yahoo.com

For computational solutions of convex optimization problems, a rich body of knowledge including theory, algorithms, and computational experience is now available. In contrast, nothing of comparable depth and completeness can be offered at the present time, for non-convex problems. The field of convex optimization benefited immensely from pre-existing body of concepts and knowledge from pure mathematics, while non-convex problems seems to require formulation and exploration of entirely new mathematical concepts, as well as new models of computation. The intent of this paper is to describe our efforts in this direction, at a philosophical or conceptual level, without going into specific applications or implementation in software. We also point out connections with other areas, particularly mathematical physics.

B2 - December 16, 17:00 – 17:50

Topology and Geometry of Amorphous Structures

Yasuaki Hiraoka
Kyushu University, Japan
hiraoka@imi.kyushu-u.ac.jp

Description of amorphous structures has been a long-standing problem. What is lacking there is an appropriate language to describe geometric structures including short-range order (SRO) and medium-range order (MRO). In this talk, we present new computational topological methods to this problem: (1) persistence modules on commutative ladders, and (2) continuations of point clouds by persistence diagrams. These methods elucidate the following new geometric features in amorphous structure: (i) persistence of MRO rings in silica glasses under pressurizations, and (ii) presence of 1-parameter deformations connecting specific packing states (FCC, HCP, 5-rings, etc) in three dimensional granular packing experiments.

B2 - December 16, 18:00 – 18:25

The Classification of Homotopy Classes of Bounded Curvature Paths

José Ayala
UNAP, Chile
jayalhoff@gmail.com

A bounded curvature path is a continuously differentiable piecewise C^2 path with bounded absolute curvature that connects two points in the tangent bundle of a surface. We give necessary and sufficient conditions for two bounded curvature paths, defined in the Euclidean plane, to be in the same connected component while keeping the curvature bounded at every stage of the deformation. This work finishes a program started by Lester Dubins in 1961.

Joint work with Hyam Rubinstein (University of Melbourne, Australia).

B2 - December 17, 15:30 – 15:55

Inducing a map on homology from a correspondence

Paweł Pilarczyk
IST Austria, Austria
pawel.pilarczyk@ist.ac.at

We define the homomorphism induced in homology by a closed correspondence between topological spaces. For that purpose, we use projections from the graph of the correspondence to its domain and codomain. We focus on correspondences that naturally emerge from combinatorial approximations of continuous maps obtained either by means of rigorous numerics or by an attempt to reconstruct the map from a finite set of data points. We show that under certain assumptions, the homomorphism induced by an outer approximation of a continuous map coincides with the homomorphism induced in homology by the map itself. In particular, our results provide a generalization of the work by Mischaikow, Mrozek and Pilarczyk, published in Found. Comput. Math., Vol. 5, 2005 (pp. 199-229).

Joint work with Shaun Harker (Rutgers University, USA), Hiroshi Kokubu (Kyoto University, Japan) and Konstantin Mischaikow (Rutgers University, USA).

B2 - December 17, 16:00 – 16:25

Practical efficiency of persistent homology computations

Hubert Wagner
IST Austria, Austria
hub.wag@gmail.com

In this talk I summarize a number of algorithmic techniques that enhance the efficiency of persistent homology computations. While the existing algorithms are at least quadratic in the number of input cells in the worst case, careful optimizations yield roughly linear behavior for practical data sets. I focus on discrete Morse theoretical preprocessing, which is especially useful for cubical (e.g. image) data. Additionally, efficient variants of the standard matrix reduction algorithm are discussed.

Joint work with Uli Bauer (IST Austria), David Gunther (Télécom ParisTech), Michael Kerber (MPI Saarbrücken) and Jan Reininghaus (IST Austria).

B2 - December 17, 17:00 – 17:25

Embeddings of Simplicial Complexes – Algorithms & Combinatorics

Uli Wagner
IST Austria, Austria
uli@ist.ac.at

We survey a number of results and open questions concerning (from a combinatorialist's point of view) higher-dimensional analogues of graph planarity and crossing numbers, i.e., embeddings of finite simplicial complexes (compact polyhedra) into Euclidean space and other ambient manifolds.

While embeddings are a classical topic in geometric topology, here we focus rather on algorithmic and combinatorial aspects. Two typical questions are the following:

(1) Is there an algorithm that, given as input a finite k-dimensional simplical complex, decides whether it embeds in d-dimensional space?

(2) What is the maximum number of k-dimensional simplices of a simplicial complex that embeds into d-dimensional space?

Time permitting, we will also discuss some maps with more general restrictions on the set of singularities, e.g., maps without r-fold intersection points.

Joint work with Martin Cadek (Masaryk University, Brno), Xavier Goaoc (Universite Paris-Est), Marek Krcal (IST Austria), Isaac Mabillard (IST Austria), Jiri Matousek (Charles University, Prague), Pavel Patak (Charles University, Prague), Zusana Safernova (Charles University, Prague), Francis Sergeraert (Institut Fourier, Grenoble), Eric Sedgwick (DePaul University, Chicago), Martin Tancer (IST Austria) and Lukas Vokrinek (Masaryk University, Brno).

B2 - December 17, 17:30 – 17:55

Parameterised complexity in 3-manifold topology

Benjamin Burton
The University of Queensland, Australia
bab@maths.uq.edu.au

Decision problems on 3-manifolds are notoriously difficult: the mere existence of an algorithm is often a major result, even "simple" problems have best-known algorithms that are exponential time, and many important algorithms have yet to be studied from a complexity viewpoint. In practice, however, some of these algorithms run surprisingly well in experimental settings. In this talk we discuss why parameterised complexity now looks to be the "right" tool to explain this behaviour. We present encouraging initial results on the parameterised complexity of topological problems, including both fixed-parameter-tractability and W[P]-hardness results, and discuss current directions of ongoing research.

Joint work with Rodney Downey (Victoria University of Wellington, New Zealand), Thomas Lewiner (PUC University of Rio de Janeiro, Brazil), João Paixão (PUC University of Rio de Janeiro, Brazil), William Pettersson (The University of Queensland, Australia) and Jonathan Spreer (The University of Queensland, Australia).

Workshop B3
Continuous Optimization

Organizers: Coralia Cartis – Pablo Parrillo – Javier Peña

B3 - December 15, 14:30 – 15:00

Randomized Methods for Zeroth-Order Optimization

Alexander Rakhlin
University of Pennsylvania, USA
rakhlin@gmail.com

We present several methods for zeroth order stochastic convex optimization and analyze their complexity. The proposed algorithms are based on random walks on convex bodies. We find that such methods can deal with the noisy information in a more stable manner.

Joint work with A. Belloni (Duke University, USA), T. Liang (University of Pennsylvania, USA) and H. Narayanan (University of Washington, USA).

B3 - December 15, 15:00 – 15:30

A Trust Region Algorithm with a Worst-Case Global Function Evaluation Complexity of $\mathcal{O}(\epsilon^{-3/2})$ for Nonconvex Smooth Optimization

Frank E. Curtis
Lehigh University, United States
frank.e.curtis@gmail.com

We present a trust region algorithm for solving nonconvex optimization problems that, in the worst-case, is able to drive the norm of the gradient of the objective below a prescribed threshold $\epsilon > 0$ after at most $\mathcal{O}(\epsilon^{-3/2})$ function evaluations, gradient evaluations, or iterations. Our work has been inspired by the recently proposed Adaptive Regularisation framework using Cubics (i.e., the ARC algorithm), which attains the same worst-case complexity bound. Our algorithm is modeled after a traditional trust region algorithm, but employs modified step acceptance criteria and a novel trust region updating mechanism that allows it to achieve this desirable property. Importantly, our method also maintains standard global and fast local convergence guarantees.

Joint work with Daniel P. Robinson (Johns Hopkins University) and Mohammadreza Samadi (Lehigh University).

B3 - December 15, 15:35 – 16:25

Classical unconstrained optimization based on "occasionally accurate" random models

Katya Scheinberg
Lehigh University, USA
katyascheinberg@gmail.com

We will present a very general framework for unconstrained optimization which includes methods using random models for deterministic and stochastic optimization. We make assumptions on the stochasticity that are different from the typical assumptions of stochastic and simulation-based optimization. In particular we assume that our models, search directions and function values satisfy some good quality conditions with some probability, but can be arbitrarily bad otherwise. Recently several convergence and expected convergence rates results have been developed under this framework when applied to standard optimization methods, such as line search, trust region method, direct search methods and adaptive regularization with cubics. We will present these results and outline the general analysis techniques based on theory of stochastic processes.

Joint work with A. Bandeira, C. Cartis, R. Chen, M. Menickelly and L.N. Vicente.

B3 - December 15, 17:00 – 17:30

Lipschitzean Piecewise Smooth Minimization (LiPSMin)

Andreas Griewank
Humboldt University, Germany
griewank@math.hu-berlin.de

We present a new method for the minimization of continuous piecewise smooth objectives. It is based on successive piecewise linearization in abs-normal form combined with quadratic overestimation. The inner problems are solved in by a bundle method in finitely many steps, and the outer iterates converge with a linear rate that depends on the curvature of the selection functions. We present numerical results on the usual test problems. Related successive piecewise linearization strategies are being developed for equation solving and the numerical integration of dynamical systems with Lipschitzian right hand sides. This is a fundamental shift of paradigm from the customary local approximation of problem functions by linear or quadratic Taylor expansions.

Joint work with Andrea Walther (University of Paderborn, Germany), Sabrina Fiege (University of Paderborn, Germany) and Torsten Bosse (Argonne National Laboratory, USA).

B3 - December 15, 17:30 – 18:00

Geodesic distance maximization via convex optimization

Maryam Fazel
University of Washington, USA
mfazel@uw.edu

Given a graph with fixed edge weights, finding the shortest path, also known as the geodesic, between two nodes is a well-studied network flow problem. We introduce the Geodesic Distance Maximization Problem (GDMP), i.e., the problem of finding the edge weights that maximize the length of the geodesic, subject to convex constraints on the weights.

We show that GDMP is a convex optimization problem for a wide class of flow costs, and provide a physical interpretation using the dual. We present applications in various fields, including network interdiction, optical lens design, and control of forest fires. GDMP can be generalized from graphs to continuous fields, where the Eikonal equation (a fundamental partial differential equation governing flow propagation) naturally arises in the dual problem. For the case of propagation on a regular grid, the problem can be cast as a second-order cone program; however standard solvers fail to scale to the large

grid sizes of interest. We develop an Alternating Direction Method of Multipliers (ADMM) algorithm by exploiting specific problem structure to solve large-scale GDMP, and demonstrate its effectiveness in numerical examples.

Joint work with De Meng (University of Washington, USA), Pablo A. Parrilo (MIT, USA) and Stephen P. Boyd (Stanford University, USA).

B3 - December 15, 18:00 – 18:30

Relative Entropy Relaxations for Signomial Optimization

Venkat Chandrasekaran
California Institute of Technology, United States of America
venkatc@caltech.edu

Signomial programs (SPs) are optimization problems consisting of an objective and constraints specified by signomials, which are sums of exponentials of linear functionals of a decision variable. SPs are non-convex optimization problems in general, and instances of NP-hard problems can be reduced to SPs. We describe a hierarchy of convex relaxations to obtain successively tighter lower bounds of the optimal value in SPs. This sequence of lower bounds is computed by solving increasingly larger-sized relative entropy optimization problems, which are convex programs specified in terms of linear and relative entropy functions. Our approach relies crucially on the observation that the relative entropy function – by virtue of its joint convexity with respect to both arguments – provides a convex parametrization of certain sets of globally nonnegative signomials with efficiently computable nonnegativity certificates via the arithmetic-geometric-mean (AM/GM) inequality. By appealing to representation theorems from real algebraic geometry, we demonstrate that our sequence of lower bounds converges to the global optimum for broad classes of SPs. Finally, we discuss numerical experiments that demonstrate the effectiveness of our approach.

Joint work with Parikshit Shah (University of Wisconsin at Madison, United States of America).

B3 - December 16, 14:35 – 15:25

Analysis and Design of Optimization Algorithms via Integral Quadratic Constraints

Benjamin Recht
University of California, Berkeley, USA
brecht@berkeley.edu

I will present a new method to analyze and design iterative optimization algorithms built on the framework of Integral Quadratic Constraints (IQC) from robust control theory. IQCs provide sufficient conditions for the stability of complicated interconnected systems, and these conditions can be checked by semidefinite programming. I will discuss how to adapt IQC theory to study optimization algorithms, proving new inequalities about convex functions. Using these inequalities, I will derive upper bounds on convergence rates for the gradient method, the heavy-ball method, Nesterov's accelerated method, and related variants by solving small, simple semidefinite programming problems. I will close with a discussion of how these techniques can be used to search for optimization problems with desired performance characteristics, establishing a new methodology for algorithm design.

Joint work with Laurent Lessard (University of California, Berkeley) and Andrew Packard (University of California, Berkeley).

B3 - December 16, 15:30 – 16:00

On the graphical derivative of solution maps to parameterized equilibria with conic constraints

Héctor Ramírez
Universidad de Chile, Chile
hramirez@dim.uchile.cl

In this talk we present new calculations of the graphical derivative for the solution map to parameterized generalized equations/KKT systems associated with conic constraints. We first compute new second-order generalized differential constructions based on the graphical derivative of the normal cone mapping appearing in the KKT system. These computations are derived provided the feasible set appearing in the KKT system is convex. They provide verifiable conditions for isolated calmness of the corresponding solution map. Then, the application of a “dilatation” technique permitted to extend this computation to the nonconvex case. The latter requires, however, an additional condition of geometric nature imposed on the considered cone. This is related to the σ-term associated with projection onto this cone and has a local character. Under this condition our formula for the graphical derivative has the same form as the formula resulting in VI over polyehdral sets, and so, it can be viewed as its generalization to a class of nonpolyhedral cones. The main results obtained in this general conic programming setting are specified for and illustrated by the second-order cone programming.

Joint work with Boris Mordukhovich (Wayne University, USA) and Jiri Outrata (Czech Academic of Sciences, Czech Republic).

B3 - December 16, 16:00 – 16:30

Integral Geometry and Phase Transitions in Conic Optimization

Martin Lotz
The University of Manchester, United Kingdom
martin.lotz@manchester.ac.uk

Integral geometry and geometric probability, going back to the work of Blaschke and Santaló, deal with measures on spaces of geometric objects, and can answer questions about the probability that random geometric objects intersect. This talk discusses various applications of (spherical) integral geometry in optimization: from the complexity theory of conic optimization to the analysis of convex approaches to solving inverse problems. In particular, it is shown how integral geometry naturally gives rise to a complete explanation of phase transition phenomena for the applicability of convex regularization to data recovery problems. We also propose extensions of the theory that allow to precisely study the probability distribution of singular values and condition numbers of conically restricted operators, and discuss relations to approaches based on geometric functional analysis.

Joint work with Dennis Amelunxen (City University Hong Kong), Michael B. McCoy (Cofacet), Joel A. Tropp (Caltech).

B3 - December 16, 17:00 – 17:30

Communication-Efficient Distributed Dual Coordinate Ascent

Martin Takac
Lehigh University, USA
takac.mt@gmail.com

Communication remains the most significant bottleneck in the performance of distributed optimization algorithms for large-scale machine learning. In this paper, we propose a communication-efficient framework, CoCoA, that uses local computation in a primal-dual setting to dramatically reduce the amount of necessary communication. We provide a strong convergence rate analysis for this class of algorithms, as well as experiments on real-world distributed datasets with implementations in Spark. In our experiments, we find that as compared to state-of-the-art mini-batch versions of SGD and SDCA algorithms, CoCoA converges to the same .001-accurate solution quality on average 25x as quickly.

Joint work with Martin Jaggi (ETH Zurich), Virginia Smith (UC Berkeley, USA), Jonathan Terhorst (UC Berkeley, USA), Sanjay Krishnan (UC Berkeley, USA), Thomas Hofmann (ETH Zurich) and Michael I. Jordan (UC Berkeley, USA).

B3 - December 16, 17:30 – 18:00

Stochastic Dual Coordinate Ascent with Arbitrary Sampling of Coordinates

Peter Richtarik
University of Edinburgh, United Kingdom
peter.richtarik@ed.ac.uk

We design a novel stochastic dual coordinate ascent (SDCA) method for minimizing an L2-regularized empirical loss function. The method operates by updating a random **set** of coordinates of the dual problem at every iteration. We allow for an **arbitrary probability law** (sampling) to govern the choice of the set of coordinates to be updated in an iteration and show how this enters the complexity bound. By varying the sampling we obtain SDCA with importance sampling, mini-batch SDCA and distributed SDCA as special cases. in a statistically interesting regime for the choice of the regularization parameter, we obtain a linear speedup up to the square root of the number of coordinates (examples). However, our method also enjoys further **data-dependent speedup.** Lastly, unlike traditional analysis of SDCA, in our analysis we control the decrease of the duality gap directly.

Joint work with Zheng Qu (Edinburgh), Tong Zhang (Baidu).

B3 - December 16, 18:00 – 18:30

A short proof of infeasibility and generating all infeasible semidefinite programs

Gabor Pataki
University of North Carolina at Chapel Hill, USA
gabor@unc.edu

The fundamental Farkas lemma of linear programming allows us to easily verify the infeasibility of an LP. For semidefinite programs (SDPs) the straightforward generalization of Farkas' lemma is not *exact*, as it does not always prove infeasibility.

We present a short and exact certificate of infeasibility of SDPs using a standard form reformulation. From the reformulation the infeasibility is easy to read off, and it allows us to systematically generate all infeasible SDPs. We prove that – loosely speaking – there are only finitely many representative infeasible SDPs in every dimension. The URL of the paper is http://arxiv.org/abs/1406.7274

I will also recall related earlier work: we call a feasible semidefinite system *well behaved* if it has strong duality with its dual for all objective functions. I will present an exact characterization of well behaved systems, which allows to systematically generate all of them.

In particular, we can systematically generate all linear maps under which the image of the semidefinite cone is closed: this is a problem of independent interest in convex geometry. The URL of the latter paper is: http://arxiv.org/abs/1112.1436

Joint work with Minghui Liu (University of North Carolina at Chapel Hill).

B3 - December 17, 14:30 – 15:00

Projection: A Unified Approach to Semi-Infinite Linear Programs with applications to Convex Optimization

Amitabh Basu
Johns Hopkins University, United States of America
basu.amitabh@jhu.edu

We extend Fourier-Motzkin elimination to semi-infinite linear programs. Applying projection leads to new characterizations of important properties for primal-dual pairs of semi-infinite programs such as zero duality gap. Our approach yields a new classification of variables that is used to determine the existence of duality gaps. Our approach has interesting applications in finite-dimensional convex optimization, such as completely new proofs of Slater's condition for strong duality.

Joint work with Kipp Martin (University of Chicago, USA) and Chris Ryan (University of Chicago, USA).

B3 - December 17, 15:00 – 15:30

Toward a Broader View of Theory of Computing – Part 3

Narendra Karmarkar
Indian Institute of Technology, Bombay, India
narendrakarmarkar@yahoo.com

Beginning with the projectively invariant method for linear programming, interior point methods have led to powerful algorithms for many difficult computing problems, in combinatorial optimization, logic, number theory and non-convex optimization. Algorithms for convex optimization benefitted from many pre-established ideas from classical mathematics, but non-convex problems require new concepts. This three part series outlines some of these concepts

In this session, we extend the Riemannian geometry underlying projective method for linear programming to more general metric g(x), to solve non-convex optimization problems. A number of computational methods in optimization and solving equations use locally approximate linear or quadratic models based on Taylor expansion of the functions at a point . We introduce a method of creating locally exact models by defining appropriate space curvature adapted to the function(s) under consideration. We introduce concepts of degree and algebraicity relative to a metric or an affine connection. If all covariant derivatives of the function except for a finite number vanish, zero set of the function is considered to be algebraic w.r.t. that affine connection. The metric is adapted to the input instance in such a way as to make the objective function "relatively algebraic" with low degree

A property of more fundamental significance than convexity is path-wise connectivity of the level sets of the objective function. We introduce graded version of this property. A connected subset is (k, l) – connected if any pair of points in the set can be joined by segment of a curve of degree not exceeding k, dimension of the affine space spanned by points on the segment not exceeding l, and lying entirely in the subset. Convex sets have the simplest type of connectivity – they are (1,1) – connected .A simple example of non-convex set is the set of m by n matrices of rank not exceeding k. This set is (2,3) – connected. Sets of higher connectivity indices are important in going beyond convex optimization. In case a curved space is introduced as above, the degree is relative to the affine connection and dimension is that of the geodesic submanifold spanned by points on the segment. Geodesically convex sets have (1,1)-connectivity relative to the metric.

B3 - December 17, 15:30 – 16:00

A New Primal-Dual Predictor-Corrector Interior-Point Method

Daniel Robinson
Johns Hopkins University, USA
daniel.p.robinson@gmail.com

I present a new primal-dual predictor-corrector interior-point method for solving nonconvex optimization problems. The method is based on a new primal-dual shifted barrier function that uses Lagrange multiplier estimates to expand the feasible region for each subproblem. This expansion of the feasible region allows for improved numerical performance. Akin to predictor-corrector methods for linear programming, the step computation involves the combination of an aggressive superlinearly convergent step with a more conservative step aimed at minimizing the merit function.

Joint work with Philip E. Gill (University of California, San Diego) and Vyacheslav Kungurtsev (Czech Technical University in Prague).

B3 - December 17, 16:00 – 16:30

Gordon's Inequality and condition numbers in conic optimization

Dennis Amelunxen
City University of Hong Kong, SAR of China
damelunx@cityu.edu.hk

The probabilistic analysis of condition numbers has traditionally been approached from different angles; one is based on Smale's program in complexity theory and features integral geometry, while the other is motivated by geometric functional analysis and makes use of the theory of Gaussian processes, notably through Slepian's and Gordon's Inequalities. In this talk we aim at providing a unifying viewpoint on these approaches, and we will showcase how the different methods can be combined. Among other things, we will introduce the concept of conically restricted linear operators, whose associated "spectrum" provides a fresh light on conic condition numbers and intriguing new conjectures about their probabilistic behavior.

Joint work with Martin Lotz (The University of Manchester, UK).

B3 - December 17, 17:00 – 17:30

Stable polynomials, matroids, and sums of squares

Cynthia Vinzant
North Carolina State University, United States
vinzant@umich.edu

In 2004, Choe, Oxley, Sokal and Wagner established a tight connection between matroids and multiaffine real stable polynomials. Recently, Brändén used this theory and a polynomial coming from the Vámos matroid to disprove the generalized Lax conjecture. I will discuss the fascinating connections between these fields and how sums of squares can be used to test both for real stability and for determinantal representability of polynomials.

B3 - December 17, 17:30 – 18:00

A certificate for non-negativity of polynomials over unbounded sets

Juan C Vera
Tilburg University, Netherlands
j.c.veralizcano@tilburguniversity.nl

Certificates of non-negativity are fundamental tools in optimization. Recently, sum-of-square certificates for non-negativity of polynomials over semialgebraic sets, have been used to obtain powerful numerical techniques to address the solution of polynomial optimization problems. Usually these certificates (e.g. Schmudgen and Putinar Positivstellensatz) require the semialgebraic set to be compact.

We present a new certificate of non-negativity for polynomials that no require the semialgebraic set to be bounded. We use this certificate to generalize classical results regarding the non-negativity of quadratic polynomials over sets defined by a quadratic. We also use it to obtain a convergent hierarchy of linear matrix inequalities for polynomial optimization problems with unbounded feasible sets.

Joint work with Javier Pena (Carnegie Mellon, USA) and Luis Zuluaga (Lehigh, USA).

B3 - December 17, 18:00 – 18:30

Nonnegative polynomials and sums of squares on real projective varieties.

Mauricio Velasco
Universidad de los Andes, Colombia
mvelasco@uniandes.edu.co

A fundamental problem in polynomial optimization is the determination of the set of polynomials of degree d in n variables which are nonnegative on a given set $X \subseteq \mathbb{R}^n$. When X is basic semialgebraic, general certificates of positivity are available by the Positivestellensatz. In this talk we focus in the case when X is a real projective variety. The main contribution is that, for some varieties, we are able to give sharp bounds on the degree of the certificate. These bounds depend only on the classical algebro-geometric invariants of X.

Specifically, in the talk I will discuss the following two cases,

(1) The classification of all varieties X and integers d for which every polynomial of degree d which is nonnegative on X can be written as a sum of squares.

(2) Positivity certificates for curves. Let $X \subseteq \mathbb{P}^n$ be a totally real curve of arithmetic genus g whose real connected components are one-dimensional. If p is a nonnegative polynomial of degree $2s$ on X and

$$k \geq \max\left(\deg(X) - n + 1, \left\lfloor \frac{2g-1}{\deg(X)} \right\rfloor + 1\right)$$

then there exists a multiplier q of degree $2k$ such that pq is a sum of squares.

Joint work with Gregory Blekherman (Georgia Tech, USA) and Gregory G. Smith (Queen's University, Canada).

B3 - Poster

Semi Stochastic Gradient Descent

Jakub Konečný
University of Edinburgh
kubo.konecny@gmail.com

In this work we study the problem of minimizing the average of a large number (n) of smooth convex loss functions. We propose a new method, S2GD (Semi-Stochastic Gradient Descent), which runs for one or several epochs in each of which a single full gradient and a random number of stochastic gradients is computed, following a geometric law. The total work needed for the method to output an ε-accurate solution in expectation, measured in the number of passes over data, or equivalently, in units equivalent to the computation of a single gradient of the loss, is $O(\log(1/\varepsilon))$. This is achieved by running the method for $O(\log(1/\varepsilon))$ epochs, with a single gradient evaluation and $O(\kappa)$ stochastic gradient evaluations in each, where κ is condition number of the problem. The SVRG method of Johnson and Zhang (SVRG) arises as a special case. To illustrate our theoretical results, S2GD only needs the workload equivalent to about 2.1 full gradient evaluations to find an 10^{-6}-accurate solution for a problem with $n = 10^9$ and $\kappa = 10^3$. Furthermore, we present a minibatching scheme, which admits simple possibility of parallelism and even improves the complexity bound under certain conditions.

Joint work with Peter Richtárik.

B3 - Poster

Nearest Neighbors Methods for Support Vector Machines

Sergio Camelo Gómez
Universidad de los Andes, Colombia
sa.camelo38@uniandes.edu.co

A key issue in the practical applicability of the Support Vector Machine methodology is the identification of the support vectors in very large data sets. In this article we propose methods based on sampling and nearest neighbours that allow for an efficient implementation of an approximate solution to the classification problem. The main idea will be that under some conditions, the support vectors of the SVM problem are, with high probability, k-neighbours of the support vectors of a random subsample from the original data. This means that if a random subsample is expanded repeatedly with the k-neighbours of its support vectors, updating the support vectors at each iteration, an approximate solution of the complete data set can be obtained at low cost. We prove some theoretical results that motivate the methodology and evaluate the performance of the proposed method with different examples.

Joint work with María González-Lima (Universidad Simón Bolivar, Venezuela) and Adolfo Quiroz (Universidad de los Andes, Colombia).

B3 - Poster

Compressed Sensing of Data with Known Distribution

Mateo Díaz
Universidad de los Andes, Colombia
m.diaz565@uniandes.edu.co

Compressed sensing is a technique with many important applications. For all these applications the most important parameter is the number of measurements required for perfect recovery. In this work we are able to drastically reduce the number of required measurements by incorporating information about the distribution of the data we wish to recover. Our algorithm works by minimizing an appropriately weighted ℓ^1 norm and our main contribution is the determination of good weights.

Joint work with Mauricio Junca, Felipe Rincon, and Mauricio Velasco (Universidad de los Andes, Colombia).

Workshop B4
Geometric Integration and Computational Mechanics

Organizers: Elena Celledoni – Marlis Hochbruck – David Martin de Diego

---

B4 - December 15, 14:30 – 14:55

Post-Lie algebras in differential geometry and applications

Hans Munthe-Kaas
University of Bergen, Norway
hans.munthe-kaas@math.uib.no

Algebraic combinatorics has become a powerful tool in the study of geometric properties of differential equations, with applications in diverse areas such as control theory, stochastic differential equations, geometric numerical integration algorithms and renormalisation theory.

Pre-Lie algebras describe algebras of flat and torsion-free connections on (mostly) euclidean spaces (Vinberg 1963). This is the algebraic foundation of Butchers B-series (Butcher 1963-72) and is closely related to a Hopf algebra appearing in renormalisation theory and non-commutative geometry (Connes-Kreimer 1999). Pre-Lie structures also appear in algebraic deformation theory (Gerstenhaber 1963). Unfortunately, pre-Lie connections do only exist on very special Lie groups, such as Euclidean spaces and certain nil-potent groups, and they are therefore not applicable for the analysis of many geometric structures appearing in mechanics and gauge field theories (principal bundles).

Post-Lie algebras is a recent invention from the last decade. The differential geometric view is the algebra of a flat connection with constant torsion. This view, with applications to numerical analysis, has been explored in a series of papers by our research group (MK-Wright 2006), MK-Lundervold 2013, Lundervold-EbrahimiFard-MK 2014). The same algebraic structure (and the name post-Lie) also appears in operad theory (Vallette 2007), where it arises as a Koszul dual of a commutative trialgebra.

Post-Lie algebras is a powerful algebraic abstraction which encodes both infinitesimal and finite aspects of flows (vector fields and their analytical or numerical flows) as well as geometric aspects such as parallel transport and curvature. There are natural post-Lie structures associated with any Lie group and more generally with homogeneous spaces and Klein geometries, where the post-Lie structure describes a connection on the Atiyah Lie algebroid.

In the talk we will survey recent developments in this field, focus on some applications in numerical integration and point out some important open research areas.

Joint work with Alexander Lundervold (Bergen University College), Olivier Verdier (Bergen University College) and Kurusch Ebrahimi-Fard (ICMAT Madrid).

B4 - December 15, 15:00 – 15:25

The Butcher group is a Lie group

Geir Bogfjellmo
Norwegian University of Science and Technology, Norway
geir.bogfjellmo@math.ntnu.no

The concept of B-series, formal expansions of numerical methods for ordinary differential equations, has been an important tool for numerical analysts over the last decades.

In 1972, Butcher showed that numerical integrators which allow a B-series expansion, form an infinite-dimensional group under composition.

In 1998, the Butcher group was rediscovered by Connes and Kreimer in the context of renormalization in Quantum Field Theory. Connes and Kreimer also showed that, algebraically, the Butcher group is associated with a Lie algebra.

We show that the Butcher group is a Lie group modeled on a Fréchet space. The Lie algebra of Connes and Kreimer reappears as a dense subalgebra of the tangent space at the identity of this group.

We explore the properties of the Butcher group from the point of infinite dimensional Lie group, thus complementing the algebraic tretament by Connes and Kreimer.

Joint work with Alexander Schmeding (Norwegian University of Science and Technology).

B4 - December 15, 15:30 – 15:55

Discrete inequalities for central-difference type operators

Takayasu Matsuo
University of Tokyo, Japan
matsuo@mist.i.u-tokyo.ac.jp

One advantage of the energy-preserving methods is that sometimes the energy gives an a priori estimate for the (numerical) solution. For example, in the cubic nonlinear Schroedinger equation, the quartic energy function (the Hamiltonian) yields an estimate $\|u\|_\infty < \infty$ for all $t > 0$, with the aid of the discrete Gagliardo–Nirenberg and Sobolev inequalities.

Although such discrete inequalities have been known for the simplest forward (i.e. one-sided) finite difference operator, it remained open for more general operators including the standard central-difference operator, as far as the authors know. Accordingly, the analyses of energy-preserving methods with such operators remained open as well.

Recently, we found a unified way of establishing discrete inequalities for a certain range of central-difference type operators. In this talk, we show some results, and illustrate them through applications to some structure-preserving numerical schemes.

Joint work with Daisuke Furihata (Osaka University) and Hiroki Kojima (University of Tokyo).

B4 - December 15, 17:05 – 17:55

Symplectic Runge-Kutta methods for nonsymplectic problems

JM Sanz-Serna
Universidad Carlos III de Madrid, Spain
jmsanzserna@gmail.com

Symplectic Runge-Kutta and Partitioned Runge-Kutta methods exactly preserve quadratic first integrals (invariants of motion) of the system being integrated. While this property is often seen as a mere curiosity (it does not hold for arbitrary first integrals), it plays an important role in the computation of numerical sensitivities, optimal control theory and Lagrangian mechanics. Some widely used procedures, such as the direct method in optimal control theory and the computation of sensitivities via reverse accumulation imply hidden integrations with symplectic Partitioned Runge-Kutta schemes.

B4 - December 15, 18:00 – 18:25

Algebra and structure-preserving integrators

Charles H. Curry
Norwegian University of Science and Technology, Norway
chc5@hw.ac.uk

Since the pioneering work of Butcher, it is known that a large class of numerical integration schemes for ordinary differential equations may be encoded algebraically using trees. Order conditions, composition of schemes and other useful properties may then be studied at the algebraic level. In applications, it is often desirable that numerical schemes retain certain structural properties of the underlying equation, such as symplecticity. These constraints may be encoded at the algebraic level. In many cases the algebraic structure simplifies accordingly. We explore the algebraic encoding of such simplifications and their implications for the study of structure-preserving numerical methods.

B4 - December 16, 14:30 – 14:55

The exact discrete Lagrangian function on the Lie algebroid of a Lie groupoid

Juan Carlos Marrero
University of La Laguna, Spain
jcmarrer@ull.edu.es

In this talk, I will present a definition of the exact discrete Lagrangian function associated with a continuous regular Lagrangian function on the Lie algebroid of a Lie groupoid. Some applications on variational analysis error in this setting will also be presented. In order to define the exact discrete Lagrangian function, I will use some results on second order differential equations on the vertical bundle of a fibration. These results may be proved using classical theorems on second order differential equations.

Joint work with D Martín de Diego (ICMAT, Spain) and E Martínez (University of Zaragoza, Spain).

B4 - December 16, 15:00 – 15:25

Reduction by stages of discrete mechanical systems: a discrete Lagrange-Poincare approach

Javier Fernandez
Instituto Balseiro, Argentina
jfernand@ib.edu.ar

Discrete mechanical systems (DMS) are a type of dynamical system whose trajectories are the extrema of a discrete variational problem determined by a discrete Lagrangian function. Those trajectories provide interesting numerical integrators. Under fairly reasonable conditions symmetric DMSs can be reduced, that is, a new dynamical system can be constructed whose dynamics captures the essential features of the original one. In some cases, it is convenient to perform the reduction procedure in more than one

step, what is generically known as reduction by stages. Unfortunately, the reduced systems are usually not DMSs so that a second reduction may not be possible within the standard DMS theory.

In this talk we expand the family of DMSs to a new family, the discrete Lagrange-Poincare systems, that is closed under reduction. As a consequence, the reduction by stages can be satisfactorily performed.

B4 - December 16, 15:35 – 16:25

Geometric Numerical Integration and Computational Geometric Mechanics

Melvin Leok
University of California, San Diego, USA
mleok@math.ucsd.edu

Symmetry, and the study of invariant and equivariant objects, is a deep and unifying principle underlying a variety of mathematical fields. In particular, geometric mechanics is characterized by the application of symmetry and differential geometric techniques to Lagrangian and Hamiltonian mechanics, and geometric integration is concerned with the construction of numerical methods with geometric invariant and equivariant properties. Computational geometric mechanics blends these fields, and uses a self-consistent discretization of geometry and mechanics to systematically construct geometric structure-preserving numerical schemes.

In this talk, we will introduce a systematic method of constructing geometric integrators based on a discrete Hamilton's variational principle. This involves the construction of discrete Lagrangians that approximate Jacobi's solution to the Hamilton-Jacobi equation. Jacobi's solution can be characterized either in terms of a boundary-value problem or variationally, and these lead to shooting-based variational integrators and Galerkin variational integrators, respectively. We prove that the resulting variational integrator is order-optimal, and when spectral basis elements are used in the Galerkin formulation, one obtains geometrically convergent variational integrators.

We will also introduce the notion of a boundary Lagrangian, which is analogue of Jacobi's solution in the setting of Lagrangian PDEs. This provides the basis for developing a theory of variational error analysis for multisymplectic discretizations of Lagrangian PDEs. Equivariant approximation spaces will play an important role in the construction of geometric integrators that exhibit multimomentum conservation properties, and we will describe two approaches based on spacetime generalizations of Finite-Element Exterior Calculus, and Geodesic Finite Elements on the space of Lorentzian metrics.

B4 - December 16, 17:00 – 17:25

Solvability of Geometric Integrators for Multi-body Systems

Marin Kobilarov
Johns Hopkins University, USA
marin@jhu.edu

This work is concerned with the solvability of implicit time-stepping methods for simulating the dynamics of multi-body systems. The standard approach is to select a time-step based on desired level of accuracy and computational efficiency of integration. Implicit methods impose an additional but often overlooked requirement that the resulting nonlinear root-finding problem is solvable and has a unique solution. Motivated by empirically observed integrator failures when using large time-steps this work develops bounds on the chosen time-step which guarantee convergence of the root-finding problem solved with

Newton's method. Second-order geometric variational integrators are used as a basis for the numerical scheme due to their favorable numerical behavior. In addition to developing solvability conditions for systems described by local coordinates, this work initiates a similar discussion for Lie group integrators which are a favored choice for floating-base systems such as robotic vehicles or molecular structures.

B4 - December 16, 17:30 – 17:55

Structure preserving integration of hybrid dynamical systems and optimal control

Sigrid Leyendecker
Chair of Applied Dynamics, University of Erlangen-Nuremberg, Germany
sigrid.leyendecker@ltd.uni-erlangen.de

The optimal control of human walking movements requires simulation techniques, which handle the contact's establishing and releasing between the foot and the ground. The system's dynamics switches non-smoothly between phases with and without contact making the system hybrid.

During motion phases without switch, the direct transcription method Discrete Mechanics and Optimal Control (DMOCC) is used to transform the optimal control problem into a constrained optimisation problem. It involves a mechanical integrator based on a discrete constrained version of the Lagrange-d'Alembert principle. This integrator represents exactly the behaviour of the analytical solution concerning the consistency of momentum maps and symplecticity. To guarantee the structure preservation and the geometrical correctness during the establishing and releasing of contacts, the non-smooth problem is solved including the computation of the contact or contact release configuration as well as the contact time and force, instead of relying on a smooth approximation of the contact problem via a penalty potential.

While in a first approach, the sequence (not the switching time) in which the closing and opening of contacts follow each other is considered as known, a more general approach is the optimisation of the whole locomotion requiring a combined model including transitions between the different dynamical systems. Integer valued functions can be used to control if and when the switch to another dynamical system occurs, i.e. they permit to control the sequence and switching times of the dynamical systems. A variable time transformation allows to eliminate the integer valued functions and therefore to apply gradient based optimisation methods to approximate the mixed integer optimal control problem.

Joint work with Michael W. Koch (Chair of Applied Dynamics, University of Erlangen-Nuremberg), Maik Ringkamp (Chair of Applied Dynamics, University of Erlangen-Nuremberg) and Sina Ober-Blöbaum (Computational Dynamics and Optimal Control, Department of Mathematics, University of Paderborn).

B4 - December 16, 18:00 – 18:25

Higher order variational integrators in the optimal control of mechanical systems

Sina Ober-Blöbaum
Freie Universität Berlin, University of Paderborn (on leave), Germany
sinaob@math.upb.de

In recent years, much effort in designing numerical methods for the simulation and optimization has been put into schemes which are structure preserving. One particular class are variational integrators which are momentum preserving and symplectic. In this talk, we develop a convergence theory on high order variational integrators applied to finite-dimensional optimal control problems posed with mechanical systems.

In the first part of the talk, we derive two different kinds of high order variational integrators based on different dimensions of the underlying approximation space. While the first well-known integrator is equivalent to a symplectic partitioned Runge-Kutta method, the second integrator, denoted as symplectic Galerkin integrator, yields a method which in general, cannot be written as a standard symplectic Runge-Kutta scheme.

In the second part of the talk, we use these integrators for the discretization of optimal control problems. By analyzing the adjoint systems of the optimal control problem and its discretized counterpart, we prove that for these particular integrators dualization and discretization commute. This property guarantees that the convergence rates are preserved for the adjoint system which is also referred to as the Covector Mapping Principle.

Joint work with Cédric M. Campos (Universidad de Valladolid, Spain).

B4 - December 17, 14:30 – 14:55

Geometric Integration for High Fidelity Visual Computing Applications

Dominik L. Michels
Stanford University, CS Dept., USA
michels@cs.stanford.edu

To be able to take into account a multitude of physical effects, high fidelity simulations are nowadays of growing interest for analysing and synthesising visual data. In contrast to most numerical simulations in engineering, local accuracy is secondary to the global visual plausibility. Global accuracy can be achieved by preserving the geometric nature and physical quantities of the simulated systems for which reason geometric integration algorithms like symplectic methods are often considered as a natural choice. Additionally, if the underlying phenomena behaves numerically stiff, a non-geometric nature comes into play requiring for strategies to capture different timescales accurately. In this contribution, a hybrid semi-analytical, semi-numerical Gautschi-type exponential integrator for modeling and design applications is presented. It is based on the idea to handle strong forces through analytical expressions to allow for long-term stability in stiff cases. By using an appropriate set of analytical filter functions, this explicit scheme is symplectic as well as time-reversible. It is further parallelizable exploiting the power of up-to-date hardware. To demonstrate its applicability in the field of visual computing, various examples including collision scenarios and molecular modeling are presented.

B4 - December 17, 15:00 – 15:25

On a problem in quantum control with unknown initial conditions

Antonella Zanna
University of Bergen, Norway
anto@math.uib.no

We consider the problem of controlling solutions of the linear Schroedinger equation using linear lasers. The initial condition is not exactly known, but it is known that it can be generated within a range of initial conditions. Is it possible to construct a laser that drives all these initial conditions to a detectable wave? We formulate the problem mathematically and discuss a numerical method to approximate the laser. This problem is relevant in the experimental physics setting to detect whether antimatter is subject to the same gravitational force as matter or not.

Joint work with Jan Petter Hansen (University of Bergen).

B4 - December 17, 15:30 – 15:55

Simulation of Wind Instruments and a Geometric Invariance of the Discrete Gradient Method

Takaharu Yaguchi
Kobe University, Japan
yaguchi@pearl.kobe-u.ac.jp

In this talk we consider simulation of wind instruments by using the Webster equation. The Webster equation is a model equation of sound waves in tubes such as vocal tracts and bodies of wind instruments. Simulation of sound waves requires long-time calculation compared to the time scale of wave propagation phenomena and hence we need structure-preserving methods to obtain meaningful results.

We apply the discrete gradient method to this equation. Because a gradient is defined by using an inner product, we must introduce a suitable Riemannian structure to the phase space. We used two different inner products to design numerical schemes by the discrete gradient method; however, it turned out that the schemes do not depend on the choice of the inner product.

By extending this result, we show a theorem that states a geometric invariance of the discrete gradient method under the change of the Riemannian structure.

Joint work with Ai Ishikawa (Kobe University).

B4 - December 17, 16:00 – 16:25

Asymptotic exponential splitting for the linear, time-dependant Schrödinger equation

Karolina Kropielnicka
University of Gdańsk, Poland
karolina.kropielnicka@mat.ug.edu.pl

This talk is about recent results obtained in numerical approximation of linear, time-dependant Schrödinger equation. The main difficulty lies in a minute size of Planck constant and in the time-dependant potential. The outcome of our investigation is an asymptotic exponential splitting, which features two significant advantages: it separates scales of the frequency and every successive term in this splitting is of increasingly higher order. This means that the accuracy of proposed methods scales linearly with their cost. These results were obtained, working at a level of differential operators, by combining Magnus expansion, the sBCH formula and Zassenhaus splitting. Additionally, to render the method more efficient, we are following the Munthe-Kaas – Owren approach of changing the basis at the stage of Magnus expansion.

Joint work with Arieh Iserles (University of Cambridge, UK) and Pranav Singh (University of Cambridge, UK).

B4 - December 17, 17:00 – 17:25

Energy preservation for moving mesh PDEs

Brynjulf Owren
Norwegian University of Science and Technology, Norway
bryn@math.ntnu.no

Recently, there has been a growing interest in designing integral preserving schemes for PDEs which use well-known ideas from ordinary differential equations, such as discrete gradient methods and the averaged vector field method. Although adapting such schemes to simple finite difference or finite element methods on constant uniform grids is straightforward, the situation becomes much more challenging when the spatial mesh is non-uniform or even changing with time. In the latter case it is not even very clear what should be meant by an integral being preserved. In this talk we shall look at various possible ways of giving meaning to the concept of integral preservation of moving mesh PDEs and we provide some promising numerical results.

Joint work with Sølve Eidnes (Norwegian University of Science and Technology).

B4 - December 17, 17:30 – 17:55

Geometric data assimilation: a thermostat-based particle filter

Jason Frank
Utrecht University, Netherlands
j.e.frank@uu.nl

Particle filters used in data assimilation are nonintrusive in the sense that they do not alter the trajectories of individual ensemble members. For this reason they offer a simple means of preserving geometric features of the dynamics to effect 'data assimilation on manifolds'. However they suffer from degeneracy of the ensemble, in which all of the ensemble weight gets assigned to a single sample, and have no mechanism for correcting ensemble drift. Thermostats have traditionally been used to perturb trajectories of molecular gases to ergodically sample equilibrium (Gibbs) measures. However they can be easily implemented to preserve additional (e.g. structural) invariants. Recent experience with thermostats also indicates they can be used to sample a nonstationary measures conditioned on data, making them a potential compromise for particle filtering.

Joint work with Keith Myerscough (CWI, Amsterdam) and Ben Leimkuhler (U. Edinburgh).

B4 - December 17, 18:00 – 18:25

Collocation method for solving singular ODEs and higher index DAEs

Ewa B. Weinmüller
Vienna University of Technology, Austria
e.weinmueller@tuwien.ac.at

During recent years, a lot of scientific work concentrated on the analysis and numerical treatment of boundary value problems (BVP) in ordinary differential equations (ODEs) which can exhibit singularities. Such problems have often the following form:

$$y'(t) = \frac{1}{t^\alpha}M(t)y(t) + f(t, y(t)), \quad t \in (0,1], \quad b(y(0), y(1)) = 0.$$

For $\alpha = 1$ the problem is called singular with a singularity of the first kind, for $\alpha > 1$ it is essentially singular (singularity of the second kind).

The search for efficient numerical methods to solve the above BVP is strongly motivated by numerous applications from physics, chemistry, mechanics, ecology, or economy. In particular, problems posed on infinite intervals are frequently transformed to a finite domain taking above form with $\alpha > 1$. Also, research activities in related fields, like differential algebraic equations (DAEs) or singular Sturm-Liouville eigenvalue problems benefit from techniques developed for singular BVPs.

While collocation stays robust and shows advantageous convergence properties in context of singular problems, other high order methods suffer from instabilities and order reductions. It turns out that for singular BVPs with smooth solutions, the convergence order of the polynomial collocation is at least equal to the so-called stage order of the method. For collocation at equidistant points or Gaussian points this convergence result means that the scheme with m inner collocation points constitutes a high order basic solver whose global error is $O(h^m)$ uniformly in t [5].

Clearly, in order to solve an ODE system efficiently, the error estimate and the mesh adaptation strategy have to be provided to correctly reflect the solution behavior. Due to the robustness of collocation, this method was used in one of the best established standard FORTRAN codes for (regular) BVPs, COLSYS [1], as well as in Matlab codes bvp4c [7], the standard module for (regular) ODEs with an option for singular problems, BVP SOLVER [8], sbvp [2], and bvpsuite [6]. The scope of bvpsuite includes fully implicit form of the ODE system with multi-point boundary conditions, arbitrary mixed order of the differential equations including zero, module for dealing with infinite intervals, module for eigenvalue problems, free parameters, and a path-following strategy for parameter-dependent problems with turning points.

We will illustrate how bvpsuite can be used to solve BVPs from applications, Complex Ginzburg-Landau equations, density profile in hydrodynamics, and generalized Korteweg-de Vries equation [3].

Finally, we turn to higher index DAEs. Higher index DAEs constitute a really challenging class of problems due to the involved differentiation which is a critical operation to carry out numerically. A possible technique to master the problem is to pre-handle the DAE system in such a way that the transformed problem is of index one and less difficult to solve. Since this approach is technically involved, it is worth to try to avoid it and provide a method which can be applied directly to the original DAE system of high index. At present, there are only some experimental results available, but they are quite encouraging and therefore, we shall briefly discuss them during the presentation [4].

References

[1] U. Ascher, J. Christiansen, and R. Russell, Collocation software for boundary value ODEs. ACM Transactions on Mathematical Software, 1981, pp. 209–222.

[2] W. Auzinger, G. Kneisl, O. Koch, and E.B. Weinmüller, A collocation code for boundary value problems in ordinary differential equations. Numer. Algorithms, 2003, pp. 27–39. Code available from http://www.mathworks.com/matlabcentral/fileexchange > Mathematics > Differential Equations > SBVP1.0 Package.

[3] C. Budd, O. Koch, S. Schirnhofer, and E.B. Weinmüller, Work in progress, 2014.

[4] M. Hanke, R. März, C. Tischendorf, E.B. Weinmüller, and S. Wurm, Work in progress, 2014.

[5] F. de Hoog and R. Weiss, Collocation Methods for Singular BVPs. SIAM J. Numer. Anal., 1978, pp. 198–217.

[6] G. Kitzhofer, O. Koch, G. Pulverer, C. Simon, and E.B. Weinmüller, Numerical Treatment of Singular BVPs: The New Matlab Code bvpsuite. JNAIAM J. Numer. Anal. Indust. Appl. Math., 2010, pp. 113–134.

[7] L. Shampine, J. Kierzenka, and M. Reichelt, Solving Boundary Value Problems for Ordinary Differential Equations in Matlab with bvp4c. 2000.

[8] L. Shampine, P. Muir, and H. Xu, A User-Friendly Fortran BVP Solver. JNAIAM J. Numer. Anal. Indust. Appl. Math., 2006, pp. 201–217.

B4 - December 17, 18:30 – 18:55

Modified trigonometric integrators

Ari Stern
Washington University in St. Louis, EEUU
astern@math.wustl.edu

Geometric integration of highly oscillatory Hamiltonian systems is difficult, due to the simultaneous presence of fast and slow time scales. Classic methods (such as Störmer/Verlet) are inefficient, since they must take extremely small time steps in order to remain stable. Trigonometric integrators represent a major advance – but even for these methods, there is a fundamental trade-off between stability and consistency with respect to certain multiscale dynamical features (slow energy exchange, near-preservation of adiabatic invariants, etc.). In this talk, we show that modified trigonometric integrators, i.e., trigonometric integrators with modified frequency, are able to sidestep this tradeoff, achieving both stability and consistency. Specifically, we prove that an implicit-explicit (IMEX) method is the unique modified trigonometric integrator which is both stable and multiscale structure-preserving, and we illustrate its performance through numerical experiments on the Fermi-Pasta-Ulam problem.

B4 - Poster

Some results on invariant measures of reduced discrete mechanical systems

Nicolás Borda
Depto. de Matemática (FCE), Universidad Nacional de La Plata - CONICET, Argentina
nborda@mate.unlp.edu.ar

The interest in discrete (time) mechanical systems is highly motivated by the construction of structure preserving numerical integrators for the continuous ones [MW01]. For instance, one of the features of their discrete evolution consists in the invariance of a Liouville measure, which is associated to an invariant canonical symplectic form on the discrete phase space; the latter is formed by pairs of positions in the configuration space (a differentiable manifold).

Another known fact about discrete mechanical systems is the momentum preservation in the presence of symmetries. Moreover, there is a well developed discrete counterpart of reduction theory of continuous systems. When passing to a dynamical system on a quotient space in order to remove the symmetries, it is worth saying that the latter result in hamiltonian systems on a Poisson manifold. In this sense, they preserve a Poisson bracket but not necessarily a symplectic form, and, thus, it is not guaranteed the existence of an invariant measure as in the previous case; the situation is analogue for discrete systems.

When satisfied, the property of unimodularity of a Poisson manifold gives an affirmative answer to the question of whether such a reduced continuous system preserves a measure. This is due to the equivalence between the notion of unimodularity and the existence of a volume form that is invariant by all hamiltonian flows [Wei97,FGM13].

It is the purpose of this work to study sufficient conditions to relate the concept of unimodulatiry of a Poisson manifold to the existence of invariant measures of reduced discrete mechanical systems. To this end, we firstly address the problem for the discrete Euler-Poincaré equations under quite general

conditions. These equations describe the reduced dynamics of discrete systems whose configuration space, G, is simultaneously a group of symmetries acting on the discrete phase space, $G\times G$, by the diagonal action induced by the left multiplication [BS99]. Reinterpreting the idea of unimodularity as the unimodularity of $(G \times G)/G \cong G$ as a Lie group [Koz88], we use results from Lie algebra theory to show that this is sufficient to prove the invariance of a certain measure.

Finally, towards a more general result, we tackle the problem of existence of invariant measures of reduced discrete mechanical systems by means of geometric arguments, instead of dealing with their equations of motion. These take into account the presevation of the reduced Poisson structure on the reduced discrete space and, also, the symplectic leaves associated to it. Although this work-in-progress technique requires stronger hypotheses when it is applied and compared to the previous particular situation, it is intended to cover a wider range of systems. As future work, our next step will be to extend the current approach to include (nonholonomic) constraints.

[BS99] A. I. Bobenko and Y. B. Suris. Discrete lagrangian reduction, discrete Euler-Poincaré equations, and semidirect products. Lett. Math. Phys., 49(1):79-93, 1999.

[FGM13] Y. N. Fedorov, L. C. García-Naranjo and J. C. Marrero. Unimodularity and preservation of volumes in nonholonomic mechanics. Preprint, arXiv:1304.1788, 2013.

[Koz88] V. V. Kozlov. Invariant measures of the Euler-Poincaré equations on Lie algebras. Funktsional. Anal. i Prilozhen., 22(1):69-70, 1988.

[MW01] J. E. Marsden and M. West. Discrete mechanics and variational integrators. Acta Numer., 10:357-514, 2001.

[Wei97] Alan Weinstein. The modular automorphism group of a Poisson manifold. J. Geom. Phys., 23:379-394, 1997.

Joint work with Javier Fernández (Instituto Balseiro, Universidad Nacional de Cuyo - CNEA, Argentina) and Marcela Zuccalli (Depto. de Matemática (FCE), Universidad Nacional de La Plata, Argentina).

Workshop B5
Information Based Complexity

Organizers: Stefan Heinrich – Aicke Hinrichs

B5 - December 15, 14:30 – 15:00

The ANOVA decomposition of a non-smooth function of an infinite number of variables

Ian Sloan
University of New South Wales, Australia
i.sloan@unsw.edu.au

In this work we extend our earlier work motivated by path-dependent option pricing problems, in which we tried to understand how it is that sparse grid and QMC methods can be applied successfully to option pricing problems, even though the integrands do not live in any mixed derivative smoothness class. That difficulty derives from the "max function" in the integrand, describing the fact that options are considered worthless if the payoff falls below the strike price.

In a previous paper (Math. Comp. 82, 383-400, 2013) we showed that if the expected value is expressed as an integral over $\mathbb{R}^d$ then the classical ANOVA decomposition of the integrand for an arithmetic Asian option can have every term smooth except for the very highest term. That highest ANOVA term has many discontinuities in first partial derivatives, but in most cases is expected to be pointwise very small.

In the present work we consider the ANOVA decomposition of the corresponding continuous problem in the Brownian bridge (or Levy-Ciesielski) formulation, and show that in this case **every term in the (infinite) ANOVA decomposition is smooth.** With this result we are preparing for an error analysis of the cubature problem for option pricing problem, in which the discrete-time problem is approximated by the continuous problem, and the error analysis then applied to the truncated infinite ANOVA expansion, in which every term is smooth.

Joint work with Frances Kuo (UNSW), Michael Griebel (Bonn).

B5 - December 15, 15:00 – 15:30

On equivalence of anchored and ANOVA spaces of multivariate functions

Grzegorz Wasilkowski
University of Kentucky, USA
greg@cs.uky.edu

We consider anchored and ANOVA spaces of functions with mixed first order partial derivatives bounded in L_1 and L_∞ norms. The spaces are weighted and we provide necessary and sufficient conditions so that the corresponding spaces have equivalent norms with constants independent of the number of variables. This includes the case of countably many variables.

B5 - December 15, 15:30 – 16:00

L_p-spaces in the anchored and ANOVA setting

Aicke Hinrichs
Johannes Kepler University Linz, Austria
aicke.hinrichs@jku.at

We extend the recent investigation of G. Wasilkowski on the equivalence of weighted anchored and ANOVA L_1 and L_∞ norms of function spaces with mixed partial derivatives of order one. Among other norms, we consider L_p norms for $1 \leq p \leq \infty$ and confirm the conjecture that for product weights, summability of the weight sequence is necessary and sufficient.

Joint work with Jan Schneider (University of Rostock, Germany).

B5 - December 15, 16:00 – 16:30

Integration w.r.t. the Standard Gaussian Measure on the Sequence Space

Mario Hefter
TU Kaiserslautern, Germany
hefter@mathematik.uni-kl.de

We consider a quadrature problem on the sequence space $\mathbb{R}^{\mathbb{N}}$, where the underlying measure $\mu = N(0,1)^{\mathbb{N}}$ is given by the countable product of standard normal distributions and the integrands $f : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ belong to a unit ball of a reproducing kernel Hilbert space. This space has tensor product form, and the building blocks are weighted Sobolev spaces of once differentiable functions where the function itself and the derivative have bounded L^2-norm with respect to the centered normal distribution $N(0,\sigma^2)$ with variance σ^2.

We consider deterministic algorithms in the worst-case-setting, where the cost of evaluating a function at a point is the index of the highest nonzero component. Upper and lower bounds for the complexity are derived.

B5 - December 15, 17:00 – 17:30

Tractability of the Approximation of High-Dimensional Rank One Tensors

Erich Novak
Jena University , Germany
erich.novak@uni-jena.de

We study the approximation of high-dimensional rank one tensors using point evaluations. We prove that for certain parameters (smoothness and norm of the rth derivative) this problem is intractable while for other parameters the problem is tractable and the complexity is only polynomial in the dimension. We completely characterize the set of parameters that lead to easy or difficult problems, respectively. In the "difficult" case we modify the class to obtain a tractable problem: The problem gets tractable with a polynomial (in the dimension) complexity if the support of the function is not too small.

Joint work with Daniel Rudolf (Jena University, Germany).

B5 - December 15, 17:30 – 18:00

On the complexity of scalar first order PDEs

Stefan Heinrich
University of Kaiserslautern, Germany
heinrich@informatik.uni-kl.de

Within the framework of information-based complexity theory, we study the approximate solution of certain classes of scalar first order partial differential equations, both in the deterministic and the randomized setting. We consider standard information. The analysis is based on the classical method of characteristics and on recent results by Th. Daun and the author on the complexity of parametric ordinary differential equations. We also discuss the case of first order PDEs depending on parameters.

B5 - December 15, 18:00 – 18:30

Tractability of approximation of ridge functions

Jan Vybiral
Department of Mathematical Analysis, Charles University, Prague, Czech Republic
vybiral@karlin.mff.cuni.cz

We give an overview of recent results on approximation of ridge functions $f(x) = g(a \cdot x)$. As the class of ridge functions is non-linear, number of interesting questions appear. We present, and analyse, several algorithms for recovery of such functions, study their numerical performance and their optimality. Surprisingly, nearly all sorts of tractability appear when trying to approximate ridge functions in high dimension from a limited number of its function values.

Joint work with Ingrid Daubechies (Duke), Massimo Fornasier (TU Munich), Anton Kolleck (TU Berlin), Sebastian Mayer (Uni Bonn), Karin Schnass (Uni Innsbruck) and Tino Ullrich (Uni Bonn).

B5 - December 15, 18:30 – 19:00

Automatic bounding of cross–derivatives

Hernan E. Leovey
Humboldt University of Berlin, Germany
leovey@math.hu-berlin.de

Calculating optimal weights for construction of rank-1 lattice rules for high dimensional integration problems in weighted spaces requires in general knowledge of the (weighted) norm of the functions at hand. The latter norm usually needs evaluation of mixed partial derivatives w.r.t. each variable at most once (cross–derivatives) in high dimensions.
Consider a function $f : \Omega \to \mathbb{R}^d$ that has cross–derivatives that are continuous over some convex compact set $\Omega \subset \mathbb{R}^d$. As for d up to about 20, evaluating all 2^d cross–derivatives in a single point $x \in \Omega$ is still quite feasible, but for much larger dimensions this task becomes prohibitively expensive. A new method for automatic bounding of cross–derivatives at a relative cost of $O(d^2)$ is given, where d is the real dimension of the problem. In the new method we are content with the computation of $2\,d + 1$ product and order dependent (POD) bounding coefficients Λ_0, $\overline{\Lambda} := (\Lambda_1, \ldots, \Lambda_d)$ and $\overline{\lambda} := (\lambda_1, \ldots, \lambda_d)$, such that for $\mathbf{i} \subset \{1, 2, \ldots, d\}$ we have $|f_{\mathbf{i}}(x)| \le |\mathbf{i}|!\,\Lambda_{|\mathbf{i}|} \prod_{j \in \mathbf{i}} \lambda_j$.
This method delivers an effective way of calculating recently introduced POD weights by minimizing a less effective integration error upper bound.

Joint work with Andreas Griewank (Humboldt University of Berlin).

B5 - December 16, 14:35 – 15:25

Numerical Integration

Josef Dick
The University of New South Wales, Australia
josef.dick@unsw.edu.au

In this talk we discuss various results on numerical integration for functions defined on a domain $D \subseteq \mathbb{R}^d$. In quasi-Monte Carlo theory one considers the domain $[0,1]^s$. In this case one wants quadrature points which are well distributed. If the dimension is large this can be a challenging problem. Another problem which often occurs is that the domain is not the unit cube (a standard example is $\mathbb{R}^s$). In this case one can either use some transformation to obtain an integral over the unit cube or construct quadrature rules in the given domain. Applications of such integration techniques include option pricing, the estimation of the expectation value of solutions of PDEs with random coefficients and some problems from machine learning.

Joint work with Christoph Aistleitner (JKU Linz, Austria), Johann Brauchart (TU Graz, Austria), Takashi Goda (University of Tokyo, Japan), Peter Kritzer (JKU Linz, Austria), Frances Kuo (UNSW, Australia), Gunther Leobacher (JKU Linz, Austria), Quoc Thong Le Gia (UNSW, Australia), Dirk Nuyens (KU Leuven, Belgium), Friedrich Pillichshammer (JKU Linz, Austria), Christoph Schwab (ETH Zürich, Switzerland), Kosuke Suzuki (University of Tokyo, Japan), Takehito Yoshiki (University of Tokyo, Japan) and Houying Zhu (UNSW, Australia).

B5 - December 16, 15:30 – 16:00

The weighted star discrepancy of Korobov's p-sets

Friedrich Pillichshammer
Johannes Kepler University Linz, Austria
friedrich.pillichshammer@jku.at

Tractability properties of various notions of discrepancy have been intensively studied in the last decade. In this presentation we consider the so-called weighted star discrepancy which was introduced by Sloan and Woźniakowski in 1998. We give an overview of existence results and constructions of point sets which can achieve polynomial or even strong polynomial tractability for suitable choices of weights. Then, for prime numbers p, we consider Korobov's p-sets:

$P_{p,s} = \{\boldsymbol{x}_0, \ldots, \boldsymbol{x}_{p-1}\}$ with

$$\boldsymbol{x}_n = \left(\left\{ \frac{n}{p} \right\}, \left\{ \frac{n^2}{p} \right\}, \ldots, \left\{ \frac{n^s}{p} \right\} \right) \quad \text{for} \quad n = 0, 1, \ldots, p-1,$$

$Q_{p^2,s} = \{\boldsymbol{x}_0, \ldots, \boldsymbol{x}_{p^2-1}\}$ with

$$\boldsymbol{x}_n = \left(\left\{ \frac{n}{p^2} \right\}, \left\{ \frac{n^2}{p^2} \right\}, \ldots, \left\{ \frac{n^s}{p^2} \right\} \right) \quad \text{for} \quad n = 0, 1, \ldots, p^2-1,$$

and $R_{p^2,s} = \{\boldsymbol{x}_{a,k} \ : \ a,k \in \{0,\dots,p-1\}\}$ with

$$\boldsymbol{x}_{a,k} = \left(\left\{ \frac{k}{p} \right\}, \left\{ \frac{ak}{p} \right\}, \dots, \left\{ \frac{a^{s-1}k}{p} \right\} \right) \quad \text{for} \ \ a,k = 0,1,\dots,p-1,$$

and prove bounds on their weighted star discrepancy which hold for any choice of weights. For product weights we give conditions under which the discrepancy bounds are independent of the dimension s. This implies strong polynomial tractability for the weighted star discrepancy. We also show that a very weak condition on the product weights suffices to achieve polynomial tractability.

Joint work with Josef Dick (UNSW Sydney, Australia).

B5 - December 16, 16:00 – 16:30

Numerical integration of functions with mixed smoothness

Mario Ullrich
Friedrich Schiller University Jena, Germany
mario.ullrich@uni-jena.de

We consider numerical integration of d-dimensional functions in (Besov and Triebel-Lizorkin) spaces $A^s_{p,\theta}$ of dominating mixed smoothness on the unit cube. For functions with zero boundary condition we prove that a (equally weighted) cubature rule, as proposed by Frolov in 1976, has the optimal order of convergence if $p,\theta \in (0,\infty)$ and $s > \max\{1/p, 1/\theta\}$, where s is the smoothness parameter. In particular, this implies the optimal order of convergence in Sobolev spaces of mixed smoothness in the same range of the parameters, which generalizes the previously known results to $p < 2$ and non-integer smoothness. In the region of "small smoothness", $s \in (1/p, 1/\theta]$, we prove an upper bound that is expected to be optimal. This bound for Triebel-Lizorkin spaces was only known for $d = 2$. By standard modifications of the algorithm we obtain the same results for functions without boundary conditions.

Joint work with Tino Ullrich (Hausdorff Center for Mathematics, Germany).

B5 - December 16, 17:00 – 17:30

Preasymptotic estimates for approximation of multivariate Sobolev functions

Thomas Kühn
Universität Leipzig, Germany
kuehn@math.uni-leipzig.de

The talk is concerned with optimal linear approximation of functions in isotropic periodic Sobolev spaces $H^s(\mathbb{T}^d)$ of fractional smoothness $s > 0$ on the d-dimensional torus, where the error is measured in the L_2-norm. The asymptotic rate – up to multiplicative constants – of the approximation numbers is well known. For any fixed dimension $d \in \mathbb{N}$ and smoothness $s > 0$ one has

$$(\star) \qquad a_n(I_d : H^s(\mathbb{T}^d) \to L_2(\mathbb{T}^d)) \sim n^{-s/d} \qquad \text{as} \quad n \to \infty\,.$$

In the language of IBC, the n-th approximation number $a_n(I_d)$ is nothing but the worst-case error of linear algorithms that use at most n arbitrary linear informations. Clearly, for numerical issues and questions of tractability one needs precise information on the constants that are hidden in $(\star)$, in particular their dependence on d.

For any fixed smoothness $s > 0$, the exact asymptotic behavior of the constants as $d \to \infty$ will be given in the talk. Moreover, I will present sharp two-sided estimates in the preasymptotic range, that means for 'small' n. Hereby an interesting connection to entropy numbers in finite-dimensional ℓ_p-spaces turns out to be very useful.

Joint work with Sebastian Mayer (Bonn), Winfried Sickel (Jena) and Tino Ullrich (Bonn).

B5 - December 16, 17:30 – 18:00

Optimal Approximation of Sobolev Functions in the L_2 and in the Supremum Norm

Winfried Sickel
Friedrich-Schiller-University Jena , Germany
winfried.sickel@uni-jena.de

Using tools taken from the theory of operator ideals and s-numbers, we develop a general approach to transfer estimates for L_2-approximation of Sobolev functions into results for L_∞-approximation under a detailed control of all involved constants. As illustration, we derive some results for isotropic Sobolev spaces $H^s(\mathbb{T}^d)$ and Sobolev spaces of dominating mixed smoothness $H^s_{\text{mix}}(\mathbb{T}^d)$, always equipped with natural norms. Also some comments to related questions for Besov spaces will be given.

Joint work with Fernando Cobos (Universidad Complutense de Madrid, Spain) and Thomas Kuehn (University of Leipzig, Germany).

B5 - December 16, 18:00 – 18:30

BMO and exponential Orlicz space estimate of the discrepancy function in arbitrary dimension

Lev Markhasin
University of Stuttgart, Germany
lev.markhasin@mathematik.uni-stuttgart.de

We generalize the results of Bilyk et al. on discrepancy in spaces with bounded mean oscillation and in exponential Orlicz spaces to arbitrary dimension. In particular, we use dyadic harmonic analysis to prove upper bounds of the BMO and exponential Orlicz space norms of the discrepancy function for the so-called order 2 digital nets. Such estimates play an important role as an intermediate step between the well-understood L_p bounds and the still elusive L_∞ asymptotics of the discrepancy function in arbitrary dimensions.

Joint work with Dmitriy Bilyk (University of Minnesota, USA).

B5 - December 17, 14:30 – 15:00

Tractability of Analytic Multivariate Problems

Henryk Woźniakowski
Columbia University, University of Warsaw, USA, Poland
henryk@cs.columbia.edu

For analytic multivariate problems we modify the usual concepts of tractability by replacing the pair (d, ε) by $(d, 1 + \log \varepsilon^{-1})$, where d denotes the number of variables and ε is an error threshold. It turns out that for some analytic multivariate problems we can get positive tractability results for this more demanding setting. We survey current results for multivariate integration and approximation defined over reproducing kernel Hilbert spaces. These results were obtained by J. Dick, G. Larcher, P. Kritzer, F. Kuo, F. Pillichshammer, I. Sloan, and the author.

B5 - December 17, 15:00 – 15:30

High-dimensional algorithms in weighted Hermite spaces of analytic functions

Peter Kritzer
Johannes Kepler University Linz, Austria
peter.kritzer@jku.at

We consider integration and approximation of functions in a class of Hilbert spaces of analytic functions defined on the $\mathbb{R}^s$. The functions are characterized by the property that their Hermite coefficients decay exponentially fast. The function spaces are weighted by two weight sequences. For numerical integration, we use Gauss-Hermite quadrature rules and show that the errors of our algorithms decay exponentially fast. Furthermore, we consider L_2-approximation where the algorithms use information based on either arbitrary linear functionals or function evaluations. Also in the case of approximation we obtain exponential error convergence. For given $\varepsilon > 0$, we study tractability in terms of s and $\log \varepsilon^{-1}$ and give necessary and sufficient conditions under which we achieve exponential convergence with various types of tractability.

Joint work with Christian Irrgeher (Johannes Kepler University Linz, Austria), Gunther Leobacher (Johannes Kepler University Linz, Austria), Friedrich Pillichshammer (Johannes Kepler University Linz, Austria) and Henryk Wozniakowski (Columbia University, USA; University of Warsaw, Poland).

B5 - December 17, 15:30 – 16:00

A refined classification of problems with (sub)exponential information complexity

Markus Weimar
Philipps-University Marburg, Germany
weimar@mathematik.uni-marburg.de

In the last 20 years a whole hierarchy of notions of tractability was proposed and analyzed by several authors. These notions are used to classify the computational hardness of continuous numerical problems in terms of the behavior of their information complexity $n(\varepsilon, d)$ as a function of the accuracy ε and the dimension d. By now a lot of effort was spend on either proving quantitative positive results (such as, e.g., the concrete dependence on ε and d within the well-established framework of polynomial tractability), or on qualitative negative results (which, e.g., state that a given problem suffers from the so-called curse of dimensionality). Although several weaker types of tractability were introduced recently, the theory of information-based complexity still lacks a notion which allows to quantify the exact (sub-/super-)exponential dependence of $n(\varepsilon, d)$ on both parameters ε and d.

In this talk we present the notion of (s, t)-weakly tractable problems which attempts to fill this gap. Within this framework the parameters s and t are used to quantitatively refine the huge class of polynomially intractable problems. For compact operators between Hilbert spaces we provide characterizations of (s,t)-weak tractability (w.r.t. various settings) in terms of singular values. In addition, our new notion will be illustrated by examples such as embedding problems of Sobolev spaces (as studied recently by Kühn, Sickel, and Ullrich). In particular, we complete the characterization of weak tractability for these problems.

Joint work with Pawel Siedlecki (University of Warsaw, Poland).

B5 - December 17, 16:00 – 16:30

Linear Tensor Product Problems and New Notions of Tractability

Pawel Siedlecki
University of Warsaw, Poland
psiedlecki@mimuw.edu.pl

We study the information complexity $n(\varepsilon, d)$ of linear tensor product problems in the worst and average case settings in terms of two recently introduced notions of tractability: uniform weak tractability and (s,t)-weak tractability. Here, d denotes the number of variables and ε is an error threshold. Uniform weak tractability means that the information complexity $n(\varepsilon, d)$ is not an exponential function of any positive power of ε^{-1} and/or d, whereas (s,t)-weak tractability means that the information complexity $n(\varepsilon, d)$ is not an exponential function of ε^{-s} and/or d^t. These notions allow us to refine tractability hierarchy of multivariate problems. We give necessary and sufficient conditions on these notions of tractability for homogeneous linear tensor product problems. This is also done for some nonhomogeneous linear tensor product problems in terms of their intrinsic nonhomogeneity parameters such as their regularity with respect to succesive variables or weights associated to variables and groups of variables. The work on (s,t)-weak tractability has been jointly done with Markus Weimar.

B5 - December 17, 17:05 – 17:55

Integration problems with a large or infinite number of variables

Michael Gnewuch
Christian Albrechts University Kiel, Germany
mig@informatik.uni-kiel.de

The evaluation of integrals of functions with a large or even infinite number of variables is an important task in physics, quantum chemistry and quantitative finance. If one, for instance, wants to compute the expectation of a funtional of a stochastic process, then employing a suitable series expansion of the stochastic process the problem can be transformed into the problem of calculating the integral of a function of infinitely many variables.

For the moment let us focus on infinite-dimensional integration. A valuable technique to tackle the integration problem is to use a function decomposition – the integrand is represented as an infinite sum of functions of finitely many variables. Cleverly designed algorithms, such as multilevel algorithms or multivariate decomposition methods, deal with a finite number of important summands. They balance the computational burden spent on the corresponding finite-dimensional integration sub-problems in order to minimize the computational cost for achieving a given error tolerance.

Important function decompositions are, e.g., the well-known ANOVA ("Analysis of Variance") decomposition or anchored decompositions.

It turns out one function decomposition may be more convenient for error analysis while another is more convenient for the actual design of competitive algorithms. An example is the approximation of certain classes of integrands of infinitely many variables by randomized algorithms: On the one hand the ANOVA decomposition is a valuable tool for the error analysis, but even the explicit computation of the ANOVA components of low order is infeasible (in the sense that it is more expensive than the original integration problem). On the other hand, the components of anchored decompositions can be computed efficiently and therefore they can be used to design algorithms, but the direct error analysis may be complicated.

In this talk we present optimal results for high- and infinite-dimensional integration. In some settings (depending on the class of integrands we consider, the class of algorithms we admit and the way we account for the computational cost) one can derive these results directly, and we will see situations were the results can be transfered to other settings.

Joint work with Jan Baldeaux (UTS Sydney, Australia), Josef Dick (UNSW Sydney, Australia), Mario Hefter (TU Kaiserslautern, Germany), Aicke Hinrichs (JKU Linz, Austria), and Klaus Ritter (TU Kaiserslautern, Germany).

B5 - December 17, 18:00 – 18:30

Approximation of piecewise Hölder classes from inexact information

Leszek Plaskota
University of Warsaw, Poland
leszekp@mimuw.edu.pl

We study algorithms for L^p-approximation ($1 \leq p < \infty$) of functions $f : [a,b] \to \mathbb{R}$ that are piecewise r-times continuously differentiable and $f^{(r)}$ are piecewise Hölder continuous with exponent $\rho \in (0,1]$. The singular points of f are unknown. Available information is blurred and given as $y_i = f(x_i) + e_i$, $1 \leq i \leq n$, where $|e_i| \leq \delta$. We show that a necessary and sufficient condition for the error of approximation to be at most ε is that $n \asymp \varepsilon^{-1/(r+\rho)}$ and $\delta \asymp \varepsilon$. The optimal algorithms use adaptive information. This generalizes previous results where information was exact and we had $\rho = 1$.

Joint work with Pawel Morkisz (AGH Krakow, Poland).

B5 - December 17, 18:30 – 19:00

Detecting singularities of piecewise smooth functions

Paweł Morkisz
AGH University of Science and Technology, Kraków, Poland
morkiszp@agh.edu.pl

Problems defined on spaces of piecewise smooth functions are known to be difficult as nonadaptive algorithms that work well for globally smooth functions usually fail. Such problems are even more difficult when information is in addition corrupted by noise. Then fundamental questions are what is the acceptable noise level that allows to solve a problem within given error, and what algorithms should be used? In this talk we consider the problem of detecting singular points of piecewise Hölder functions based on noisy function evaluations. The noise is assumed to be bounded. This problem is important since for many other problems like function approximation or integration one has to localize the singular points in the first place. We also provide a numerical illustration.

Joint work with Leszek Plaskota (University of Warsaw, Poland).

Workshop B6
Random Matrices

Organizers: Alan Edelman – Raj Rao

B6 - December 15, 14:30 – 14:55

Asymptotic Degrees of Freedom for Combining Regression with Factor Analysis

Patrick Perry
New York University, USA
pperry@stern.nyu.edu

In multivariate regression problems with multiple responses, there often exist unobserved covariates (features) which are correlated with the responses. It is possible to estimate these covariates via factor analytic (eigenvector) methods, but calculating unbiased error variance estimates after adjusting for latent factors requires assigning appropriate degrees of freedom to the estimated factors. Many ad-hoc solutions to this problem have been proposed without the backup of a careful theoretical analysis. Using recent results from random matrix theory, we derive an expression for degrees of freedom. Our estimate gives a principled alternative to ad-hoc approaches in common use. Extensive simulation results show excellent agreement between the proposed estimator and its theoretical value.

Joint work with Natesh Pillai (Harvard University, USA).

B6 - December 15, 15:00 – 15:25

Deformed smallest singular value laws

Brian Rider
Temple University, US
brian.rider@temple.edu

We characterize the limiting smallest eigenvalue distributions for sample covariance type matrices drawn from a spiked population in terms of random integral operators. From here we derive partial differential equations satisfied by the corresponding distribution functions. We also show that, under a natural limit, these spiked "hard edge" laws degenerate to the critically spiked Tracy-Widom laws of basic importance in mathematical statistics. As a final application we derive a dynamic characterization of the Wishart distribution (which can be viewed as a Dufresne identity for matrix processes).

Joint work with Jose Ramirez (Universidad de Costa Rica) and Benedek Valko (University of Wisconsin - Madison).

B6 - December 15, 15:35 – 16:25

Random Matrices and the Melting Polar Ice Caps

Kenneth M. Golden
Department of Mathematics, University of Utah, USA
golden@math.utah.edu

The precipitous loss of Arctic sea ice has far outpaced expert predictions. In this lecture we will delve into the mathematical underpinnings of this mystery, and discuss how we are using the mathematics of multiscale composites and statistical physics to study key sea ice properties. In particular, we will explore how random matrices arise in these problems, and show how the onset of connectedness in composite microstructures gives rise to striking transitional behavior in the long and short range correlations of the eigenvalues of the associated random matrices. This work is helping to improve projections of the fate of Earth's ice packs, and the response of polar ecosystems. We will conclude with a short video from a 2012 Antarctic expedition where sea ice properties were measured.

Joint work with N. Benjamin Murphy (Department of Mathematics, UC Irvine, USA).

B6 - December 15, 17:05 – 17:30

Hypergeometric Functions of Matrix Arguments and Linear Statistics of Multi-Spiked Hermitian Matrix Models

Matthew McKay
Hong Kong University of Science and Technology, Hong Kong
eemckay@ust.hk

This talk will present central limit theorems (CLTs) for linear statistics of three related "multi-spiked" Hermitian random matrix ensembles: (i) a spiked central Wishart ensemble; (ii) a non-central Wishart ensemble with fixed-rank non-centrality parameter; and (iii) a similarly defined non-central F ensemble. The analysis in each case is non-trivial, with the underlying joint eigenvalue densities involving hypergeometric functions of matrix arguments. For such functions, we first generalize a recent result of Onatski to present new exact multiple contour integral representations. Based on these, explicit CLT formulas are derived for each of the three spiked models of interest by employing Dyson's Coulomb Fluid method along with saddlepoint techniques. We find that for each model, the individual spikes contribute additively to yield a O(1) correction term to the asymptotic mean of the linear statistic, whilst having no effect on the leading order terms of the mean or variance.

Joint work with Damien Passemier (Hong Kong University of Science and Technology, Hong Kong), Yang Chen (University of Macau, Macau).

B6 - December 15, 17:30 – 17:55

Large Complex Correlated Wishart Matrices: Fluctuations and Asymptotic Independence at the Edges

Jamal Najim
CNRS and Université Paris Est, France
najim@univ-mlv.fr

We study the asymptotic behavior of eigenvalues of large complex correlated Wishart matrices at the edges of the limiting spectrum. For this matrix model, the support of the limiting eigenvalue distribution may have several connected components. Under mild conditions, we will show that the extremal eigenvalue which converge almost surely towards the edges of the support fluctuate according to the Tracy-Widom law at the scale $N^{2/3}$. Moreover, given several generic positive edges, we establish that the associated extremal eigenvalue fluctuations are asymptotically independent. Finally, when the leftmost edge is the origin, we prove that the smallest eigenvalue fluctuates according to the hard-edge Tracy-Widom law at

the scale N^2 (Bessel kernel). As an application, an asymptotic study of the condition number of large correlated Wishart matrices is provided.

Joint work with Walid Hachem (Telecom Paristech and CNRS, France) and Adrien Hardy (KTH - Royal Institute of Technology, Sweden).

B6 - December 16, 14:30 – 14:55

Dysonian dynamics of the Ginibre ensemble

Maciej Nowak
Jagiellonian University, Poland
nowak@th.if.uj.edu.pl

We study the time evolution of Ginibre matrices whose elements undergo Brownian motion. The non-Hermitian character of the Ginibre ensemble binds the dynamics of eigenvalues to the evolution of eigenvectors in a nontrivial way, leading to a system of coupled nonlinear equations resembling those for turbulent systems. We formulate a mathematical framework allowing simultaneous description of the flow of eigenvalues and eigenvectors, and we unravel a hidden dynamics as a function of a new complex variable, which in the standard description is treated as a regulator only. We solve the evolution equations for large matrices and demonstrate that the nonanalytic behavior of the Green's functions is associated with a shock wave stemming from a Burgers-like equation describing correlations of eigenvectors. We conjecture that the hidden dynamics that we observe for the Ginibre ensemble is a general feature of non-Hermitian random matrix models and is relevant to related physical applications.

B6 - December 16, 15:00 – 15:25

Finite N corrections to the Tracy-Widom distribution at the hard edge of the Laguerre-Wishart ensemble of complex random matrices

Grégory Schehr
University of Orsay-Paris Sud, France
gregory.schehr@u-psud.fr

We study the probability distribution function (PDF) of the smallest eigenvalue of Laguerre-Wishart matrices $W = X^\dagger X$ where X is a random $M \times N$ ($M \geq N$) matrix, with complex Gaussian independent entries. We compute this PDF in terms of semi-classical orthogonal polynomials, which can be viewed as a deformation of Laguerre polynomials. By analyzing these polynomials, and their associated recurrence relations, in the limit of large N, large M with $M/N \to 1$ – i.e. for quasi-square large matrices X – we show that this PDF can be expressed in terms of the solution of a Painlevé III equation, as found by Tracy and Widom by analyzing a Fredholm determinant built from the Bessel kernel. In addition, our method allows us to compute the first $1/N$ corrections to this limiting Tracy-Widom distribution (at the hard edge). Our computations corroborate a recent conjecture by Edelman, Guionnet and Péché.

Joint work with Anthony Perret (University of Orsay, Paris-Sud).

B6 - December 16, 15:35 – 16:25

Non-backtracking spectrum of random graphs

Charles Bordenave
University of Toulouse & CNRS, France
charles.bordenave@math.univ-toulouse.fr

The non-backtracking matrix of a graph is a non-symmetric matrix on the oriented edge of a graph which has interesting algebraic properties and appears notably in connection with the Ihara zeta function and in some generalizations of Ramanujan graphs. It has also been used recently in the context of community detection. In this talk, we will study the largest eigenvalues of this matrix for the Erdos-Renyi graph G(n,c/n) and for simple inhomogeneous random graphs (stochastic block model).

Joint work with Marc Lelarge (INRIA) and Laurent Massoulié (Microsoft & INRIA)..

B6 - December 16, 17:00 – 17:25

A Rice method proof of the NSP property for a Random matrix.

Jean-Marc Azais
Université de Toulouse, France
jean.marc.azais@gmail.com

We define the Compressed Sensing condition for exact reconstruction in a linear model with no noise under sparsity condition. We show that it is equivalent to the Null Space property: NSP. Best constructions are obtained for random matrices. For a random design matrix we show the property using random field method an obtain an explicit transition diagram.

B6 - December 17, 14:30 – 14:55

Sampling unitary ensembles

Sheehan Olver
The University of Sydney, Australia
Sheehan.Olver@sydney.edu.au

We develop a computationally efficient algorithm for sampling from a broad class of unitary random matrix ensembles that includes but goes well beyond the straightforward to sample Gaussian Unitary Ensemble (GUE). The algorithm exploits the fact that the eigenvalues of unitary ensembles (UEs) can be represented as a determinantal point process whose kernel is given in terms of orthogonal polynomials. Consequently, our algorithm can be used to sample from UEs for which the associated orthogonal polynomials can be numerically computed efficiently. By facilitating high accuracy sampling of non-classical UEs, the algorithm can aid in the experimentation-based formulation (or refutation) of universality conjectures involving eigenvalue statistics that might presently be unamenable to analysis. Examples of such experiments are included.

Joint work with Raj Rao (University of Michigan) and Thomas Trogdon (New York University).

B6 - December 17, 15:00 – 15:25

Gap probabilities and applications to geometry and random topology

Antonio Lerario
Institut Camille Jordan, Lyon, France
anto.lerario@gmail.com

What is the volume of the set of singular symmetric matrices of norm one? What is the probability that a random plane misses this set? What is the expected "topology" of the intersection of random quadric hypersurfaces? In this talk I will combine classical techniques form algebraic topology ("spectral sequences") with ideas from Random Matrix Theory and show how these problems can be solved using a local analysis of the "gap probability" at zero (the probability that a random matrix has a gap in its spectrum close to zero).

Joint work with Erik Lundberg (Florida Atlantic University).

B6 - December 17, 15:30 – 15:55

Random Matrix Laws and Jacobi Operators

Alan Edelman
MIT, USA
edelman@math.mit.edu

The four big asymptotic level density laws of Random Matrix Theory are the semicircle, the Marchenko-Pastur, the McKay, and the Wachter Laws. They correspond to the equilibrium measures for Hermite, Laguerre, Gegenbauer, and Jacobi Polynomials. The associated Jacobi matrix is Toeplitz except for first row and first column. We explore properties of these big laws, and apply the Toeplitz nature in an algorithm for the moment problem. In the second part of this talk, we consider multivariate polynomials orthogonal with respect to the product of scalar weights with the Vandermonde to the beta repulsion term.

Joint work with Alex Dubbs (MIT) and Praveen Venkataramana (MIT).

B6 - December 17, 16:00 – 16:25

New applications of random matrices

Raj Rao Nadakuditi
University of Michigan, USA
rajnrao@umich.edu

We describe some new success stories where random matrix theory has enabled new applications: these include new theory and algorithms for transmitting light perfectly through highly scattering media and for separating foreground and background of videos in highly cluttered scenes. We conclude by highlighting some newly discovered random matrix universality phenomena emerging from scattering theory and semidefinite optimization.

B6 - December 17, 17:05 – 17:55

New Formulae Relating Finite GOE and LUE — from Numerical Experiments to Proofs

Folkmar Bornemann
TU München, Germany
bornemann@tum.de

Gap probabilities of finite-dimensional GOE can be expressed as Pfaffians of single integrals with jump discontinuities that severely bound the effectiveness of quadrature-based numerical methods. Starting from recursive relations with LUE that are either valid in the large matrix limit (due to Metha and Cloizeaux) or hold for every second gap at even dimensions (due to Forrester), we have systematically explored candidates for the “missing” formulae through numerical experiments guided by heuristic arguments. While some of those candidates seem to be exact, other are still surprisingly accurate (and, therefore, probably useful) already for small dimensions even though they are only asymptotically exact. Proving the observed facts is ongoing work.

Workshop B7
Symbolic Analysis

Organizers: Evelyne Hubert – Peter Paule – Enrique Reyes

B7 - December 15, 14:30 – 14:55

A decision method for integrability of partial differential algebraic Pfaffian systems.

Lisi D'Alfonso
Universidad de Buenos Aires, Argentina
lisi@cbc.uba.ar

Let $m, n \in \mathbb{N}$. Let $x_1, \ldots, x_m$ be independent variables and $\mathbf{y} := y_1, \ldots, y_n$ be differential unknowns. For each pair (i, j), $1 \leq i \leq n$, $1 \leq j \leq m$, let f_{ij} be a polynomial in $\mathbb{C}[\mathbf{y}]$. A differential algebraic Pfaffian system is a system of differential equations as follows:

$$\Sigma = \begin{cases} \dfrac{\partial y_i}{\partial x_j} & = & f_{ij}(\mathbf{y}), \qquad \text{for } i = 1, \ldots, n \text{ and } j = 1 \ldots m, \\ \mathbf{g}(\mathbf{y}) & = & 0 \end{cases}$$

where $\mathbf{g} := g_1, \ldots, g_s$ are polynomials in $\mathbb{C}[\mathbf{y}]$.

In this work we are interested in the integrability of these systems, that is, in the existence of infinitely differentiable functions over an open set $\mathcal{U}$ of $\mathbb{C}^m$ that are solutions of Σ. The classical Frobenius Theorem (1877) establishes conditions for a Pfaffian system, without algebraic constraints, to be completely integrable. We focus on the integrability, not necessarily complete, of systems like Σ.

We associate to each system Σ a strictly decreasing chain of algebraic varieties in $\mathbb{C}^n$ of length at most $n + 1$. We prove that a necessary and sufficient condition for the existence of solutions for Σ is that the smallest variety of this chain is nonempty. From this result, we are able to show an effective procedure that allows us to decide whether a Pfaffian system is integrable in triple exponential time in n, the number of unknowns.

Joint work with Gabriela Jeronimo (Universidad de Buenos Aires) and Pablo Solernó (Universidad de Buenos Aires).

B7 - December 15, 15:00 – 15:25

Higher order integrable Lagrangians

Rafael Hernández Heredero
Universidad Politécnica de Madrid, Spain
rafahh@etsist.upm.es

We will show results on the integrability of Euler-Lagrange equations arising from Lagrangians of second order. Integrability will be understood under the symmetry approach of A. Shabat et al. Lagrangian sytems usually yield equations of hyperbolic type, and applying the symmetry approach to hyperbolic equations is quite difficult. But in the case of Euler-Lagrange equations, the symmetry approach is applicable if one uses its generalization to certain non evolutionary equations as reported in R. Hernández Heredero, A. Shabat and V. Sokolov, J. Phys. A: Math. Gen. 36 (2003) L605–L614.

B7 - December 15, 15:30 – 15:55

Symmetry classification of curvature evolutions

Peter H van der Kamp
La Trobe University, Australia
peterhvanderkamp@gmail.com

Starting from an action of a Lie-group on a manifold the Fels-Olver moving frame method provides a set of generating invariants together with their syzygies. One of the syzygies gives us the evolution of curvature invariants if the evolution of a curve is specified. Another syzygy gives us an invariant symmetry condition which we utilise to identify integrable curvature evolutions.

Joint work with Evelyne Hubert (INRIA Méditerranée, Sophia Antipolis, France).

B7 - December 15, 16:00 – 16:25

Configuration and differential invariants

David Blazquez-Sanz
Universidad Nacional de Colombia - Medellín, Colombia
dblazquezs@unal.edu.co

We study the relation between the notions of configuration and differential invariants for a G-manifold. The configuration invariants are G-invariant functions defined in the cartesian powers of the G-manifold. Configurations invariants are simpler to understand and to compute than differential invariants, since the prolongation algorithm is just the repetition of the action and does not involve derivation. There is a simpler version of the Lie-Tresse theorem for configuration invariants. We present a general framework that allows to compute the differential invariants associated to a G-manifold from its configuration invariants.

Joint work with Juan Sebastián Díaz (Universidad Nacional de Colombia - Medellín, Colombia).

B7 - December 15, 17:05 – 17:55

Discrete moving frames with applications

Elizabeth Mansfield
University of Kent, United Kingdom
e.l.mansfield@kent.ac.uk

Lie group based moving frames offer a significant new symbolic technology for the study of differential systems. By taking a sequence of frames, many of the excellent features of frames can be adapted to working with difference problems. Indeed, one can obtain a small set of generators of the algebra of invariants, and recurrence relations playing the role of differential syzygies. As for smooth frames, the relations on the difference invariants can be effectively and efficiently computed, without solving for the discrete frame.

In this talk, I will give an overview of the ideas, and show some applications to the difference calculus of variations.

Joint work with Gloria Mari Beffa (University of Madison Wisconsin, USA), Peter Hydon (University of Surrey, UK) and Linyu Peng (Waseda University, Japan).

B7 - December 15, 18:00 – 18:25

Nonlocal symmetries and formal integrability

Enrique G. Reyes
Universidad de Santiago de Chile, Chile
ereyes@fermat.usach.cl ; e_g_reyes@yahoo.ca

In this talk I introduce a simplified version of the classical Krasil'shchik-Vinogradov geometric theory of nonlocal symmetries and present several applications. For example, the theory can be used to find highly non-trivial explicit solutions and Darboux-like transforms to nonlinear equations such as the Kaup-Kupershmidt equation. I also recall the theory of formal integrability and argue that nonlocal symmetries can be used to uncover formally integrable equations. Finally, I present some classifications of nonlocal symmetries of integrable equations which have been recently found, and propose a generalization of the Krasil'shchik-Vinogradov theory.

This talk is partially based on the following papers:

1. E.G. Reyes, Geometric integrability of the Camassa-Holm equation. Letters in Mathematical Physics 59 (2002), 117–131.

2. E.G. Reyes, Nonlocal symmetries and the Kaup-Kupershmidt equation. Journal of Mathematical Physics 46 (2005), 073507 (19 pages).

3. P. Gorka and E.G. Reyes, The modified Camassa-Holm equation. International Mathematics Research Notices (2011) Vol. 2011, 2617–2649.

4. R. Hernandez-Heredero and E.G. Reyes, Geometric integrability of the Camassa-Holm equation II. International Mathematics Research Notices (2012) Vol. 2012, 3089–3125.

5. E.G. Reyes, Jet bundles, symmetries, Darboux transforms. Contemporary Mathematics 563 (2012), 137–164.

6. P. Gorka and E.G. Reyes, The modified Hunter-Saxton equation. Journal of Geometry and Physics 62 (2012), 1793–1809.

7. P.M. Bies, P. Gorka and E.G. Reyes, The dual modified KdV–Fokas–Qiao equation: geometry and local analysis. Journal of Mathematical Physics 53 (2012), 073710.

8. R. Hernandez-Heredero and E.G. Reyes, Nonlocal symmetries, compacton equations, and integrability. International Journal of Geometric Methods in Modern Physics 10 (2013), 1350046 [24 pages].

9. I.S. Krasil'shchik and A.M. Vinogradov, Nonlocal trends in the geometry of differential equations: Symmetries, conservation laws and Backlund transformations. Acta Appl. Math. 15 (1989), 161–209.

B7 - December 16, 14:30 – 14:55

Fast algorithms for the p-curvature of differential operators

Alin Bostan
INRIA, France
alin.bostan@inria.fr

The p-curvature of a linear differential operator in characteristic p is a matrix that measures to what extent the solution space of the operator has dimension close to its order. We describe a recent algorithm for computing the characteristic polynomial of the p-curvature in time $\tilde{O}(p^{0.5})$. The new algorithm allows to test the nilpotency of the p-curvature for primes p of order 10^6, for which the p-curvature itself is impossible to compute using current algorithms.

Joint work with Xavier Caruso (Université Rennes 1, France) and Éric Schost (University of Western Ontario, Canada).

B7 - December 16, 15:00 – 15:25

Computations with Nested Integrals in Particle Physics

Clemens Raab
DESY, Germany
clemens.raab@desy.de

Nested integrals over rational integrands have already been considered by Kummer and Poincaré. Slightly more general integrands involving roots arise in recent computations in the context of perturbative quantum chromodynamics. We discuss symbolic methods dealing with these nested integrals occurring in generating functions, integral transforms, and convolution integrals.

Joint work with Jakob Ablinger (RISC, Austria), Johannes Blümlein (DESY, Germany) and Carsten Schneider (RISC, Austria).

B7 - December 16, 15:35 – 16:25

Equivalence and Invariants: an Overview

Peter Olver
University of Minnesota, USA
olver@umn.edu

Two objects are said to be equivalent under a prescribed transformation group if one can be mapped to the other by a group element. In particular, symmetries of an object are just its self-equivalences. The case of submanifolds under Lie group and Lie pseudo-group actions is of particular importance, and Élie Cartan gave a general solution to the equivalence problem that relies on matching the functional interdependencies, or syzygies, among their differential invariants. Cartan's solution has been recast into the method of differential invariant signatures that has broad applicability, including image processing, differential equations, the calculus of variations, control theory, a broad range of mathematical physics, differential geometry, classical invariant theory, and so on.

The goal of this talk is to compare and contrast competing approaches to the equivalence problem and the computation of invariants, concentrating on those involving exterior differential systems, those relying on moving frames, and infinitesimal methods dating back to Lie. Recent developments, practical algorithms and some applications of interest will be mentioned during the lecture.

B7 - December 16, 17:00 – 17:25

q-shift operators in knot theory

Christoph Koutschan
RICAM, Austrian Academy of Sciences, Austria
christoph.koutschan@ricam.oeaw.ac.at

In knot theory, the colored Jones function is a knot invariant which is an infinite sequence of Laurent polynomials. Through its definition by state sums it is known to be q-holonomic, i.e., to satisfy a linear recurrence of the form $c_d f_{n+d} + \cdots + c_0 f_n = 0$, $c_d \neq 0$, whose coefficients $c_0, \ldots, c_d$ are bivariate polynomials in q and q^n. We discuss how symbolic computation supports the investigation of this knot invariant.

Joint work with Stavros Garoufalidis (Georgia Tech).

B7 - December 16, 18:00 – 18:25

Combinatorics, Number Theory, and Symbolic Analysis

Peter Paule
RISC, Johannes Kepler University, Austria
Peter.Paule@risc.uni-linz.ac.at

Partition numbers $p(n)$ give the number of additive decompositions of nonnegative integers. For example, $4 = 3+1 = 2+2 = 2+1+1 = 1+1+1+1$, so $p(4) = 5$. Ramanujan observed that all numbers $p(5n+4)$, $n \geq 0$, are divisible by 5. Recently, in the context of modular forms, Silviu Radu (RISC) has set up an algorithmic machinery to prove such congruences automatically. The talk is about new developments in this area and discusses various connections between combinatorics, number theory, and symbolic analysis.

B7 - December 17, 14:35 – 15:25

Multiple binomial sums

Bruno Salvy
INRIA, France
Bruno.Salvy@inria.fr

Multiple binomial sums form a very rich class with a lot of structure, which makes it possible to design specific algorithms that prove or discover closed forms or recurrences. In particular, these sums can be expressed as diagonals of rational functions and recurrences can then be obtained by computing a linear differential equation satisfied by the integral of a rational function. We discuss the complexity aspects of this approach as well as its practical use, and compare it to variants of Zeilberger's algorithm.

Joint work with Alin Bostan (Inria, France) and Pierre Lairez (Inria, France).

B7 - December 17, 15:30 – 15:55

Desingularization of Ore Operators

Manuel Kauers
Johannes Kepler University, Linz, Austria
manuel@kauers.de

Singularities of linear differential operators are points at which the numerical computation of solutions is cumbersome. In some cases, this is unavoidable because there is a solution which has a strange behaviour (e.g. a pole) at this point. But sometimes a singularity of a differential operator is only a "false alarm" and does not really correspond to a singularity of a solution. Such singularities are called apparent. Desingularization algorithms eliminate apparent singularities from a given operator. Such algorithms are known since the 19th century. In the talk, we will present a surprisingly simple desingularization algorithm that works not only for differential operators but for general Ore operators. This is joint work with Shaoshi Chen and Michael Singer (arXiv:1408.5512).

Joint work with Shaoshi Chen (Chinese Academy of Sciences, China) and Michael F. Singer (North Carolina State University, USA).

B7 - December 17, 16:00 – 16:25

Computing the parameterized differential Galois group of a second-order linear differential equation with parameters

Carlos E. Arreche
North Carolina State University, USA
cearrech@ncsu.edu

Consider a linear differential equation

$$\frac{\partial^2 Y}{\partial x^2} + r_1 \frac{\partial Y}{\partial x} + r_0 Y = 0,$$

where the coefficients $r_1, r_0 \in \mathbb{C}(t_1, \ldots, t_m, x)$. The parameterized Picard-Vessiot theory developed by Phyllis Cassidy and Michael Singer associates a differential Galois group G to such an equation. In analogy with the classical Picard-Vessiot theory of Kolchin, G measures the differential-algebraic relations amongst the solutions to the equation, with respect to $\frac{\partial}{\partial x}$ as well as $\frac{\partial}{\partial t_1}, \ldots, \frac{\partial}{\partial t_m}$.
Relying on earlier work by Thomas Dreyfus, I will describe a complete set of algorithms to compute G, and how these algorithms lead to a simple procedure to decide whether any of the solutions to the equation are differentially transcendental with respect to one or several of the parametric derivations $\frac{\partial}{\partial t_1}, \ldots, \frac{\partial}{\partial t_m}$.

B7 - December 17, 17:00 – 17:25

Algebraic bivariate hypergeometric Laurent series

Alicia Dickenstein
Universidad de Buenos Aires, Argentina
alidick@dm.uba.ar

I will present joint work with Eduardo Cattani and Federico Martínez on the algebricity of hypergeometric Laurent series in two variables, associated to Cayley configurations of n lattice configurations in n space. We show that these algebraic series are generated by certain combinatorially defined sums of point residues.

B7 - December 17, 17:30 – 17:55

Invariants of Finite Abelian Groups and their use in Symmetry Reduction of Dynamical Systems

George Labahn
University of Waterloo, Canada
glabahn@uwaterloo.ca

We describe the computation of rational invariants of the linear action of a finite abelian group in the non-modular case and investigate its use in symmetry reductions of dynamical and polynomial systems. Finite abelian subgroups of $GL(n, K)$ can be diagonalized which allows the group action to be accurately described by an integer matrix of exponents. We can make use of integer linear algebra to compute both a minimal generating set of invariants and the substitution to rewrite any invariant in terms of this generating set. The set of invariants provide a symmetry reduction scheme for dynamical and polynomial systems whose solution set is invariant by a finite abelian group action. A special case of the symmetry reduction algorithm applies to reduce the number of parameters in physical, chemical or biological models.

Joint work with Evelyne Hubert (INRIA Mediterranee, France).

B7 - December 17, 18:00 – 18:25

Section, Invariants and Symmetrization

Evelyne Hubert
INRIA Méditerranée, France
Evelyne.Hubert@inria.fr

For varieties invariant under the action of a finite group, it is known how to convert a set of generators for the ideal into a set of invariants with the same variety. We offer an analogue construction for algebraic groups of positive dimension.

B7 - Poster

Computing periods of rational integrals

Pierre Lairez
TU Berlin, Germany
lairez@tu-berlin.de

A period of rational integral is the result of integrating, with respect to one or several variables, a rational function along a closed path. When the period under consideration depends on a parameter, it satisfies a specific linear differential equation called Picard-Fuchs equation. These equations and their computation are important for computer algebra, but also for algebraic geometry (where they contains geometric invariants), in combinatorics (where many generating functions are periods) or in theoretical physics.

I present an efficient algorithm to compute these equations. An implementation, which involves only commutative Groebner bases and linear algebra, is available and has been successfully applied to problems that were previously out of reach.

Workshop C1
Computational Algebraic Geometry

Organizers: Carlos D'Andrea – Gregory Smith – Agnes Szanto

C1 - December 18, 14:35 – 15:25

Cactus varieties of cubic forms

Kristian Ranestad
University of Oslo, Norway
ranestad@math.uio.no

The rank of a symmetric form is the length of its shortest decomposition as a sum of pure powers of linear forms, i.e. the shortest smooth apolar scheme. The cactus rank of the form is the the length of the shortest apolar scheme. The a-th cactus variety of cubic forms $C_{a,n}$ is the closure of the family of cubic forms of cactus rank a in the projective space of cubic forms in $n+1$ variables. I shall report on recent work with Bernardi, Jelisiejew and Marques giving the dimension and a geometric characterization of the general member of $C_{a,n}$ when $1 \leq a \leq 2n+2$.

Joint work with Alessandra Bernardi (Universita de Bologna, Italy), Joachim Jelisiejew (University of Warsaw, Poland) and Pedro Macias Marques (Universidade de Evora, Portugal).

C1 - December 18, 15:30 – 15:50

An algebraic approach to phase retrieval

Cynthia Vinzant
North Carolina State University, USA
vinzant@umich.edu

The problem of phase retrieval is to reconstruct a signal from certain magnitude measurements. This problem is closely related to low-rank matrix completion and has many imaging-related applications: microscopy, optics, and diffraction imaging, among others. In purely mathematical terms, phase retrieval means recovering a complex vector from the modulus of its inner product with certain measurement vectors. One can ask how many measurements are necessary for this recovery to be possible. I'll discuss recent progress made on this problem by translating it into algebraic language and talk about my adventures as an algebraist in frame theory.

Joint work with Aldo Conca (University of Genova, Italy), Dan Edidin (University of Missouri, USA) and Milena Hering (University of Edinburgh, UK).

C1 - December 18, 16:00 – 16:20

Cellular Binomial Ideals

Laura Matusevich
Texas A&M University, USA
laura@math.tamu.edu

Without any restrictions on the base field, we compute the hull and provide an unmixed decomposition of a cellular binomial ideal. The latter had already been proved by Eisenbud and Sturmfels in characteristic zero, and conjectured to also hold in positive characteristic. Over an algebraically closed field, we further obtain an explicit (but not necessarily minimal) primary decomposition of such an ideal.

The binomial primary decomposition algorithms developed by Eisenbud and Sturmfels, and improved by Ojeda and Piedra, and Kahle, perform a cellular decomposition as a first step. We believe that our results provide further improvements to these algorithms, paving the way for the implementation of binomial primary decomposition over finite fields.

Joint work with Zekiye Sahin Eser (Texas A&M University, USA).

C1 - December 18, 17:00 – 17:20

Applications of computational algebraic geometry to vacuum moduli spaces of supersymmetric models in physics

Michael Stillman
Cornell University, USA
mike@math.cornell.edu

Given a supersymmetric potential function, such as one for the MSSM (minimal super-symmetric extension of the standard model of particle physics), there is a naturally associated affine algebraic variety, which is (essentially) the moduli space of possible vacua of the theory. In this talk we describe the structure of some of these moduli spaces, including the Electro-weak sector of the MSSM, which were obtained with the help of computational algebraic geometry and Macaulay2. We consider theories where one allows for more than 3 generations of particles. Since nature seems to have chosen 3 generations, theories for which this number of generations is forced would be ideal. Although it does not appear that 3 generations is forced here, we see how the geometry varies with different numbers of generations of particles.

We will assume no knowledge of physics during this talk. We will briefly describe the physics needed, and then we will describe the algebraic geometric and computational methods and results which allow the structure to become apparent.

This talk is based on the preprint arxiv.org/1408.6841

Joint work with Yang-Hui He (Oxford and City University London, UK), Vishnu Jejjala (University of the Witwatersrand, Johannesburg, South Africa), Cyril Matti (City University, London, UK) and Brent Nelson (Northeastern University, USA).

C1 - December 18, 17:30 – 17:50

Effective computations on Grassmann, Flag, and Stiefel varieties with applications

Chris Peterson
Colorado State University, USA
peterson@math.colostate.edu

Consider a sequence of images of a fixed object collected under a continuous variation of state (varying illumination, frequency, pose, etc). In pixel space, the collection of images can be intuitively viewed as lying on a manifold. Noise, quantization, and background clutter significantly degrade this model. What one actually sees is a data cloud clustered near the manifold. The Grassmann, flag, and Stiefel varieties have proven to be effective settings for extracting information about the underlying manifold and for comparing the appearance of different objects imaged under the same variations of state. In this setting one is led to consider problems such as "how can you average a collection of subspaces?" and "what if the subspaces are of differing dimensions?" and "what point on a given Schubert variety lies closest to a given point on a Grassmann manifold?". In this talk, we will give an overview of some of the theory and computational techniques that have allowed for solutions to these and related problems.

Joint work with Michael Kirby (Colorado State University, math), Bruce Draper (Colorado State University, computer science), Tim Marrinan (Colorado State University) and Justin Marks (Bowdoin College).

C1 - December 18, 18:00 – 18:20

Generalized barycentric coordinates and algebraic geometry

Hal Schenck
University of Illinois, USA
schenck@math.uiuc.edu

Let P_d be a convex polygon with d vertices. The associated Wachspress surface W_d is a fundamental object in approximation theory, defined as the image of the rational map w_d from $\mathbb{P}^2$ to $\mathbb{P}^{d-1}$, determined by the Wachspress barycentric coordinates for P_d. We show w_d is a regular map on a blowup X_d of $\mathbb{P}^2$, and if $d > 4$ is given by a very ample divisor on X_d, so has a smooth image W_d. We determine generators for the ideal of W_d, and prove that in graded lex order, the initial ideal of $I(W_d)$ is given by a Stanley-Reisner ideal. As a consequence, we show that the associated surface is arithmetically Cohen-Macaulay, of Castelnuovo-Mumford regularity two, and determine all the graded betti numbers of $I(W_d)$.

Joint work with Corey Irving (Santa Clara University, USA).

C1 - December 18, 18:30 – 18:50

Plethysm and lattice point counting

Thomas Kahle
Otto-von-Guericke Universität Magdeburg, Germany
thomas.kahle@ovgu.de

We show that the coefficient of the Schur functor S^λ in the decomposition of the plethysm $S^\mu(S^k)$ into irreducibles is the solution to a lattice point counting problem. Consequently, for each fixed μ, the solution to this problem is a piecewise quasi-polynomial in (λ, k). We show how to use computer algebra to determine this function explicitly when μ is a partition of 4 or 5. We also discuss asymptotics of the resulting piecewise quasi-polynomials.

Joint work with Mateusz Michalek (Simons Institute, UC Berkeley).

C1 - December 19, 14:30 – 14:50

Partitioning on varieties and point-hypersurface incidences

Martin Sombra
ICREA and University of Barcelona, Spain
sombra@ub.edu

I will describe a new polynomial partitioning result applicable to finite sets of points in a variety of codimension at most 2. It generalizes the Guth-Katz polynomial partitioning theorem as well later generalizations of this result to sets of points in hypersurfaces.

This result opens up the possibility of proving new incidence bounds in higher dimensions, and we apply it to the problem of bounding incidences between points and hypersurfaces in 4-dimensional real space.

Joint work with Saugata Basu (Purdue University, USA).

C1 - December 19, 15:00 – 15:20

Schubert varieties and distances between subspaces of different dimensions

Lek-Heng Lim
University of Chicago, USA
lekheng@galton.uchicago.edu

We resolve two fundamental problems regarding subspace distances that have arisen considerably often in applications: How could one define a notion of distance between (i) two linear subspaces of different dimensions, or (ii) two affine subspaces of the same dimension, in a way that generalizes the usual Grassmann distance between equidimensional linear subspaces? We show that (i) is the distance of a point to a Schubert variety, and (ii) is the distance within the Grassmannian of affine subspaces. In our context, a Schubert variety and the Grassmannian of affine subspaces are both regarded as subsets of the usual Grassmannian of linear subspaces. Combining (i) and (ii) yields a notion of distance between (iii) two affine subspaces of different dimensions. Aside from reducing to the usual Grassmann distance when the subspaces in (i) are equidimensional or when the affine subspaces in (ii) are linear subspaces, these distances are intrinsic and do not depend on any embedding of the Grassmannian into a larger ambient space. Furthermore, they can all be written down as concrete expressions involving principal angles, and are efficiently computable in numerically stable ways. We show that our results are largely independent of the Grassmann distance — if desired, it may be substituted by any other common distances between subspaces. Central to our approach to these problems is a concrete algebraic geometric view of the Grassmannian that parallels the differential geometric perspective that is now well-established in applied and computational mathematics. A secondary goal of this article is to demonstrate that the basic algebraic geometry of Grassmannian can be just as accessible and useful to practitioners.

Joint work with Ke Ye (University of Chicago).

C1 - December 19, 15:30 – 15:50

Algorithmic and Geometric Aspects of Sparse Decomposition

Bernard Mourrain
Inria, France
Bernard.Mourrain@inria.fr

Several problems in signal processing, tensor decomposition, sparse interpolation, polynomial optimization, etc reduce to the reconstruction of sum of exponential functions from truncated moment sequences. Prony proposed in 1795 a method to solve this problem for functions of one variable, when enough moments are known. We describe a generalization in several variables and a new algorithm to compute the decomposition of series as sums of exponential functions. An ingredient of the approach is a flat extension property, which is related to the commutation property of multiplication operators modulo a

zero-dimensional ideal. We analyze this problem from a geometric point of view, describe its connection with the Hilbert scheme of points and present some applications to reconstruction and approximation problems.

C1 - December 19, 16:00 – 16:20

A numerical algorithm for zero counting

Teresa Krick
Universidad de Buenos Aires & CONICET, Argentina
krick@dm.uba.ar

The purpose of this talk is to honor and remember our dear friend Mario Wschebor.

Together with Mario, Felipe Cucker and Gregorio Malajovich, we produce and analyze a numerical (iterative) algorithm for computing the exact number of real projective zeros of a system of n real homogeneous polynomial equations in $n+1$ variables. We first show that the number of iterations, and the cost of each iteration, depends on a condition number of the system, in addition to other usual parameters. The algorithm can be implemented with finite precision as long as the round-off unit satisfies some bound depending on the same parameters. Then we show that the condition number that we introduced satisfies an Eckard-Young theorem, as it represents the inverse of the distance of the input system to the ill-posed systems. We derive from this smoothed analysis bounds for it, applying general results obtained by Bürgisser, Cucker and Lotz. Finally, we use specific probability techniques as a Rice theorem to obtain more precise bounds for the tail and the expected value of the condition number, considered as a random variable when imposing a probability measure on the space of polynomial systems.

Joint work with Felipe Cucker (City University of Hong Kong), Gregorio Malajovich (Universidade Federal do Rio de Janeiro) and Mario Wschebor (Universidad de la República del Uruguay).

C1 - December 19, 17:00 – 17:20

Probabilistic method for toric ideals

Sonja Petrovic
Illinois Institute of Technology, USA
sonja.petrovic@iit.edu

In this talk we will discuss a probabilistic algorithm that can be used to obtain a generating set of a toric ideal. Apart from being of interest in computational algebra, the problem has applications in statistics, where toric ideal generators form Markov bases for statistical models.

Joint work with Jesús A. De Loera (University of California, Davis) and Despina Stasi (Illinois Institute of Technology).

C1 - December 19, 17:30 – 17:50

Dual toric codes and polytopes of degree one.

Mauricio Velasco
Universidad de los Andes, Colombia
mvelasco@uniandes.edu.co

A construction due to Hansen associates a linear code over a finite field $\mathbb{F}_q$ to the projective toric variety $X(P)$ specified by a lattice polytope P. The toric code V is the subspace of $\mathbb{F}_q^t$ constructed by evaluating the polynomials spanned by the lattice points of P at the t points of the torus in $X(P)$ defined over $\mathbb{F}_q$. The dual toric code is its dual subspace V^* consisting of all linear functionals in $(\mathbb{F}_q^t)^*$ which vanish identically on V.

In this talk we will show that some statistical features of the code V^* are controlled by the geometry of the toric variety $X(P)$. This correspondence allows us to distinguish extremal dual toric codes and to classify them using the classification of polytopes of degree one due to Batyrev and Nill.

Joint work with Valerie Gauthier (Universidad del Rosario, Colombia).

C1 - December 19, 18:00 – 18:20

Computing global vector fields on varieties with torus actions

Nathan Ilten
Simon Fraser University, Canada
nilten@sfu.ca

The space of global vector fields is an important invariant of an algebraic variety. In this talk, I will discuss an approach to computing this space for smooth varieties endowed with the effective action of an algebraic torus. Indeed, for such varieties, the space of global vector fields can be described in terms of global vector fields on a suitable quotient. For the special case of rational varieties with a complexity-one torus action, these computations become quite explicit.

This approach relies upon general machinery for describing equivariant vector bundles on varieties with torus action.

Joint work with Hendrik Suess (University of Edinburgh).

C1 - December 20, 14:30 – 14:50

The Maximum Likelihood Threshold of a Graph

Seth Sullivant
North Carolina State University, USA
smsulli2@ncsu.edu

The maximum likelihood threshold of a graph is the smallest number of data points that guarantees that maximum likelihood estimates exist almost surely in the Gaussian graphical model associated to the graph. We show that this graph parameter is connected to the theory of combinatorial rigidity. In particular, if the edge set of a graph G is an independent set in the $n-1$-dimensional generic rigidity matroid, then the maximum likelihood threshold of G is less than or equal to n. This connection allows us to prove many results about the maximum likelihood threshold.

Joint work with Elizabeth Gross (San Jose State University).

C1 - December 20, 15:00 – 15:20

Computing tropical curves via homotopy continuation

Josephine Yu
Georgia Tech, United States of America
jyu@math.gatech.edu

We develop a method for computing tropical curves using numerical algebraic geometry. Our method guesses rays in tropical curves by sampling points in amoebas, and we develop numerical procedures to check whether a point is in the tropical variety without having to compute any Groebner bases. We also give an implementation of our methods. As an application, we use this technique to compute Newton polygons of A-polynomials of knots.

Joint work with Anders Jensen (Aarhus University) and Anton Leykin (Georgia Tech).

C1 - December 20, 15:35 – 16:25

From chemical reaction networks to Descartes' rule of signs

Alicia Dickenstein
Universidad de Buenos Aires, Argentina
alidick@dm.uba.ar

In the context of chemical reaction networks with mass-action and other rational kinetics, a major question is to preclude or to guarantee multiple positive steady states. I will explain this motivation and I will present necessary and sufficient conditions in terms of sign vectors for the injectivity of families of polynomials maps with arbitrary real exponents defined on the positive orthant. These conditions extend existing injectivity conditions expressed in terms of Jacobian matrices and determinants, obtained by several authors. In the context of real algebraic geometry, this approach can be seen as the first partial multivariate generalization of the classical Descartes' rule, which bounds the number of positive real roots of a univariate real polynomial in terms of the number of sign variations of its coefficients. This is joint work with Stefan Müller, Elisenda Feliu, Georg Regensburger, Anne Shiu and Carsten Conradi. I will also present some further advances in this multivariate generalization obtained in collaboration with Frédéric Bihan.

C1 - December 20, 17:00 – 17:20

Newton homotopies and certification

Jonathan Hauenstein
University of Notre Dame, USA
hauenstein@nd.edu

A homotopy in which only the constant terms change is called a Newton homotopy. Such homotopies are used in moving robots, solving PDEs, and performing monodromy loops. This talk will introduce computations involving Newton homotopies, methods for a priori and a posteriori path tracking for Newton homotopies, and complexity results.

Joint work with Ian Haywood (North Carolina State University, USA) and Alan Liddell (University of Notre Dame, USA).

C1 - December 20, 17:30 – 17:50

Computing with noncommutative algebras in Macaulay2

Frank Moore
Wake Forest University, USA
moorewf@wfu.edu

NCAlgebra is a new package for Macaulay2 which provides tools for working with noncommutative graded rings and matrices over such rings. In this presentation, we give an overview of the package's primary methods, including those which compute noncommutative Gröbner bases, Hilbert series, central and normal elements, and kernels of matrices. We will also give examples of calculations that been made possible with this package.

Joint work with Andy Conner (St. Mary's University, Moraga, CA, USA).

C1 - December 20, 18:00 – 18:20

Degree Bounds in Rational Sums of Squares Representations on Curves

Greg Blekherman
Georgia Tech, USA
greg@math.gatech.edu

Let X be a curve with dense real points. It is well-known that any polynomial p nonnegative on X can be written as a sum of squares of rational functions in the coordinate ring of X. I will present new degree bounds for these rational sums of squares representations, which depend on the Hilbert series of X only. The bound can be shown to be tight in many instances. It is an open question whether the bound is tight for any curve X with dense real points.

Joint work with Greg Smith (Queens University) and Mauricio Velasco (Universidad de los Andes).

C1 - Poster

Sampling Zeros, Model Zeros, and Maximum Likelihood Degrees

Elizabeth Gross
San Jose State University, USA
elizabeth.gross@sjsu.edu

Given a statistical model, the maximum likelihood degree is the number of complex solutions to the likelihood equations for generic data, or equivalently, the degree of the likelihood locus. In this poster, we consider discrete algebraic statistical models and explore the solutions to the likelihood equations when the data are no longer generic, but instead contain zeros. In this case, with the help of numerical algebraic geometry, we see that the solutions partition into two clusters, solutions to the likelihood equations for sampling zeros and solutions that lie on the coordinate hyperplanes. Using this fact, we show how the problem of finding critical points to the likelihood function can be partitioned into smaller and computationally easier problems involving sampling and model zeros.

Joint work with Jose Rodriguez (University of Notre Dame).

C1 - Poster

Cox rings of some big rational surfaces

JOSE DARIO BASTIDAS OLAYA
UNIVERSIDAD DE LOS ANDES, COLOMBIA
jd.bastidas251@uniandes.edu.co

Smooth rational surfaces with big anticanonical class are known to have finitely generated Cox ring, but an explicit set of generators and their relations is not known. For a family of these surfaces, obtained as blow-ups of $\mathbb{P}^2$ at special point configurations, we prove that the Cox ring is generated by sections supported on negative curves. For some of these configurations we have a conjectural description of the relations. We believe they are generated by quadrics, and for a given degree we can decide computationally whether there are or there are not new relations in this degree. Some of our results, for both generators and relations, are computer-assisted proofs using Macaulay2. These results are joint work with Mauricio Velasco.

Joint work with MAURICIO VELASCO (UNIVERSIDAD DE LOS ANDES, COLOMBIA).

C1 - Poster

Algebraic geometry of tree tensor network states

Sara Jamshidi
Pennsylvania State University, USA
jamshidi@math.psu.edu

Tree tensor networks have been used to model the ground states of Hamiltonians in condensed matter physics and quantum chemistry. Exactly which quantum states can be represented by a tree tensor network with a given topology and given restrictions on the parameter tensors? When the restrictions are algebraic, the set of states is a projective algebraic variety. We describe those varieties, using techniques originally developed for phylogenetics.

Joint work with Jason Morton (Pennsylvania State University, USA).

C1 - Poster

Multiplicities of Classical Varieties

Jack Jeffries
University of Utah, United States
jeffries@math.utah.edu

The j-multiplicity plays an important role in the intersection theory of Stückrad-Vogel cycles, while recent developments confirm the connections between the ϵ-multiplicity and equisingularity theory. We establish, under some constraints, a relationship between the j-multiplicity of an ideal and the degree of its fiber cone. As a consequence, we are able to compute the j-multiplicity of all the ideals defining rational normal scrolls. By using the standard monomial theory, we can also compute the j- and ϵ-multiplicity of ideals defining determinantal varieties: The found quantities are integrals which, quite surprisingly, are central in random matrix theory. This is joint work with Jonathan Montaño and Matteo Varbaro.

Joint work with Jonathan Montaño (Purdue University, USA) and Matteo Varbaro (University of Genoa, Italy).

Workshop C2
Foundation of Numerical PDE's

Organizers: Ricardo Durán – Shi Jin – Ricardo Nochetto

C2 - December 18, 14:30 – 15:05

Numerical approximation of the time-harmonic Maxwell system using H1-conforming finite elements

Andrea Bonito
Texas A&M University , USA
bonito@math.tamu.edu

We describe a new approximation technique for the Maxwell eigenvalue problem based on H1-conforming finite elements. While reviewing the relevant properties of the Maxwell operator, we point out the difficulties for H1-conforming finite element methods to produce correct spectral approximations. It turns out that the key idea consists of controlling the divergence of the electric field in a fractional Sobolev space with differentiability index between -1 and -1/2. To illustrate the essence of our method, we first examine a non-implementable scheme with this property. Its implementable version relying on a lagrange multiplier to impose such control on the divergence is then discussed. Finally, we examine the case of heterogeneous media. In this context the method needs to cope, in addition, with electric fields not much more than square integrable.

C2 - December 18, 15:10 – 15:45

Scattering of transient waves by penetrable obstacles

Francisco-Javier Sayas
University of Delaware, USA
fjsayas@math.udel.edu

n this talk we will present some results on models for the scattering of transient linear waves by different types of obstacles: homogeneous isotropic; non-homogeneous; elastic. The coupled systems will be discretized with BEM or BEM-FEM in space, and with some implicit or implicit-explicit time-stepping method. We will discuss general stability properties, how to obtain convergence estimates for the fully discrete problems, as well as some delicate questions on how the stability constants behave as functions of time. The results collect current and previous work with Lehel Banjai, Christian Lubich, Tianyu Qiu, Tonatiuh Sanchez-Vizuet, and Matthew Hassell.

C2 - December 18, 15:50 – 16:25

A posteriori error estimators for weighted norms. Adaptivity for point sources and local errors

Pedro Morin
Instituto de Matemática Aplicada del Litoral, Universidad Nacional del Litoral, Argentina
pmorin@santafe-conicet.gov.ar

We develop a posteriori error estimates for general second order elliptic problems with point sources in two- and three-dimensional domains. We prove a global upper bound and a local lower bound for the error measured in a weighted Sobolev space. The weight belongs to the Muckenhoupt's class A2. The purpose of the weight is twofold. On the one hand it weakens the norm around the singularity, and on the other hand it strengthens the norm in a region of interest, to obtain localized estimates. The theory hinges on local approximation properties of either Clement or Scott-Zhang interpolation operators, without need of suitable modifications, and makes use of weighted estimates for fractional integrals and maximal functions. Numerical experiments illustrate the excellent performance of an adaptive algorithm with the obtained error estimators.

C2 - December 18, 17:05 – 17:55

Finite element spectral approximation of the curl operator in multiply connected domains

Rodolfo Rodriguez
Departamento de Ingenieria Matematica, Universidad de Concepcion, Chile
rodolfo@ing-mat.udec.cl

A couple of numerical methods based on Nedelec finite elements have been recently introduced and analyzed in [1] to solve the eigenvalue problem for the curl operator in simply connected domains. This topological assumption is not just a technicality, since the eigenvalue problem is ill-posed on multiply connected domains, in the sense that its spectrum is the whole complex plane as is shown in [2]. However, additional constraints can be added in order to recover a well posed problem with a discrete spectrum [2,3]. We choose as additional constraint a zero-flux condition of the curl on all the cutting surfaces. We introduce two weak formulations of the corresponding problem, which are convenient variations of those studied in [1]; one of them is mixed and the other a Maxwell-like formulation. We prove that both are well posed and show how to modify the finite element discretization from [1] to take care of these additional constraints. We prove spectral convergence of both discretizations and establish a priori error estimates. We also report numerical tests which allow assessing the performance of the proposed methods.

[1] R. Rodriguez and P. Venegas, Numerical approximation of the spectrum of the curl operator, Math. Comp. (online: S 0025-5718(2013)02745-7).

[2] Z. Yoshida and Y. Giga, Remarks on spectra of operator rot, Math. Z., 204 (1990) 235–245.

[3] R. Hiptmair, P.R. Kotiuga and S. Tordeux, Self-adjoint curl operators, Ann. Mat. Pura Appl., 191 (2012) 431–457.

C2 - December 19, 14:30 – 15:05

Stability of an upwind Petrov-Galerkin discretization of convection diffusion equations

Snorre Harald Christiansen
Center of Mathematics for Applications at University of Oslo, Norway
snorrec@math.uio.no

We study a numerical method for convection diffusion equations, in the regime of small viscosity. We identify a norm for which we have both continuity and an inf-sup condition, which are uniform in mesh-width and viscosity, up to logarithmic terms, as long as the viscosity is smaller that the mesh-width. The analysis allows for the formation of a boundary layer.

C2 - December 19, 15:10 – 15:45

Norms in the analysis of the DPG method with optimal test functions

Norbert Heuer
Pontificia Universidad Catolica de Chile, Chile
nheuer@mat.puc.cl

Standard analysis of the discontinuous Petrov-Galerkin method (DPG) with optimal test functions is based on a direct relationship between trial and test spaces, and their norms. Depending on the particular problem under consideration, theoretical and practical requirements imply different conditions both for the selection of spaces and for the definition of norms. In this talk, we discuss several cases (like convection-dominated diffusion, non-conforming trace approximation, and hypersingular boundary integral operators) and show how problem-dependent objectives force the selection of norms

Joint work with Leszek Demkowicz (The University of Texas at Austin, USA), Michael Karkulik (Pontificia Universidad Catolica de Chile) and Francisco-Javier Sayas (University of Delaware, USA).

C2 - December 19, 15:50 – 16:25

Numerical analysis of electrorheological fluids

Lars Diening
LMU Munich, Institute of Mathematics, Theresienstr. 39, 80333 Munich, Germany
diening@math.lmu.de

We study a priori estimates for the finite element solutions of electrorheological fluids. These fluids may change their viscosity significantly if an electrical field is applied. Different from the Newtonian case (like water), the friction depends non-linearly on the symmetric gradient of the velocity (power law ansatz). In the case of electrorheological fluids this power is depending additionally on the applied electrical field. We explain step by step the difficulties of the numerical analysis. We start with the p-Laplace system, continue with the p-Stokes system and finally discuss the system for electrorheological fluids. The last step requires the use of Lebesgue spaces with variable exponents.

C2 - December 19, 17:05 – 17:55

The f-wave propagation algorithm for hyperbolic PDEs

Randall J. LeVeque
Applied Mathematics Department University of Washington, USA
rjl@amath.washington.edu

Finite volume methods for solving hyperbolic PDEs, including nonlinear conservation laws whose solutions contain shock waves, are often based on solving one-dimensional Riemann problems. The wave-propagation algorithms implemented in the Clawpack software package provide a very general and robust approach to defining high-resolution methods that exhibit second-order accuracy in smooth regions of the solution while avoiding nonphysical oscillations near discontinuities. This approach is easily applied also to linear hyperbolic problems that are not in conservation form, and can be extended to two or

three space dimensions by the introduction of "transverse Riemann solvers". These algorithms have also been generalized to the so-called f-wave formulation, in which the flux difference between adjacent cells is decomposed as a linear combination of eigenvectors of suitable flux Jacobian matrices. This approach has advantages in many applications including nonlinear problems with spatially varying flux functions, which arise for example in nonlinear elasticity problems in heterogeneous media. The f-wave approach also allows incorporating source terms directly into the Riemann solver in a natural manner, which is essential for some balance laws where the solution sought is a small perturbation to a nontrivial steady state in which nonzero source terms balance the divergence of the flux. Using the shallow water equations to model the propagation of a tsunami across the ocean is an example where this approach has been critical.

C2 - December 20, 14:30 – 15:05

Quadrilateral Q_k elements and the regular decomposition property

Gabriel Acosta
University of Buenos Aires, Argentina
gacosta@dm.uba.ar

Let $K \subset \mathbb{R}^2$ be a convex quadrilateral. In [1] the following definition can be found: K satisfies the regular decomposition property with constants $N < \infty$ and $0 < \psi < \pi$ if we can divide K into two triangles along one of its diagonals, called d_1, in such a way that $|d_2|/|d_1| < N$ and the maximum angle of both triangles is bounded by ψ. Moreover, in [1] it is shown that the constant in the estimate of the H_1 norm of the error for the Q_1-Lagrange interpolation can be bounded in terms of N and ψ. In [2] this result is generalized to $W^{1,p}$ for $1 \leq p < 3$, while for $3 \leq p$ it is shown that the constant can be bounded in terms of the minimal and the maximal angle of K. In this talk we show the role of the regular decomposition property in quadrilateral Q_k interpolation for $k \geq 2$.

[1] G. Acosta, R. Duran Error estimates for Q_1-isoparametric elements satisfying a weak angle condition. SIAM J. Numer. Anal. 38, 1073-1088, 2000.

[2] G. Acosta, G. Monzon Interpolation error estimates in $W^{1,p}$ for degenerate Q_1-isoparametric elements. Numer. Math. , 104, pp 129-150, 2006.

Joint work with Gabriel Monzón (Universidad de General Sarmiento, Argentina).

C2 - December 20, 15:10 – 15:45

Multi-dimensional polynomial interpolation on arbitrary nodes

Dongbin Xiu
University of Utah, USA
dongbin.xiu@utah.edu

Polynomial interpolation is well understood on the real line. In multi-dimensional spaces, one often adopts a well established one-dimensional method and fills up the space using certain tensor product rule. Examples like this include full tensor construction and sparse grids construction. This approach typically results in fast growth of the total number of interpolation nodes and certain fixed geometrical structure of the nodal sets. This imposes difficulties for practical applications, where obtaining function values at a large number of nodes is infeasible. Also, one often has function data from nodal locations that are not by "mathematical design" and are "unstructured". In this talk, we present a mathematical framework for conducting polynomial interpolation in multiple dimensions using arbitrary set of unstructured nodes. The resulting method, least orthogonal interpolation, is rigorous and has a straightforward numerical implementation. It can faithfully interpolate any function data on any nodal sets, even on those that are considered singular by the traditional methods. We also present a strategy to choose "optimal" nodes that result in robust interpolation. The strategy is based on optimization of Lebesgue function and has certain highly desirable mathematical properties.

C2 - December 20, 15:50 – 16:25

Asymptotic-preserving and well-balanced uncertainty quantification for kinetic and hyperbolic equations

Shi Jin
University of Wisconsin-Madison, USA
jin@math.wisc.edu

In this talk we will study the generalized polynomial chaos (gPC) approach to hyperbolic and kinetic equations with uncertain coefficients/inputs, and multiple time or space scales, and show that they can be made asymptotic-preserving or well-balanced, in the sense that the gPC scheme preserves various asymptotic limits in the discrete space. This allows the implemention of the gPC methods for these problems without numerically resolving (by space, time, and gPC modes) the small scales.

Workshop C3
Learning Theory

Organizers: Tomaso Poggio – Lorenzo Rosasco

C3 - December 18, 14:30 – 15:00

A Tale of Three Regression Problems

Alexander Rakhlin
University of Pennsylvania, USA
rakhlin@wharton.upenn.edu

We consider the problem of regression in three scenarios: (a) random design under the assumption that the model F is correctly specified, (b) distribution-free statistical learning with respect to a reference class F; and (c) online regression with no assumption on the generative process. The first problem is often studied in the literature on nonparametric estimation, the second falls within the purview of statistical learning theory, and the third is studied within the online learning community. Do these three problems really differ from the minimax point of view? This question will be addressed in this talk.

Joint work with Karthik Sridharan (Cornell University) and Sasha Tsybakov (ENSAE-Paris Tech).

C3 - December 18, 15:00 – 15:30

Stochastic proximal methods for online learning

Silvia Villa
Istituto Italiano di Tecnologia, Italia
silvia.villa@iit.it

In this talk I will present recent advances on the convergence properties of a class of stochas- tic proximal gradient algorithms for solving minimization problems. These algorithms are easy to implement and suitable for solving high dimensional problems thanks to the low memory requirement of each iteration. Moreover, they are particularly suitable for compos- ite optimization, where a convex objective function is the sum of a smooth and a non-smooth component. I will show that this algorithm can be naturally applied to solve standard online machine learning algorithms and I will focus on convergence in expectation and convergence almost surely of the iterates.

C3 - December 18, 15:35 – 16:25

Efficient minimax strategies for online prediction

Peter Bartlett
UC Berkeley and Queensland University of Technology, USA
peter@berkeley.edu

Consider prediction games in which, in each round, a strategy makes a decision, then observes an outcome and pays a loss. The aim is to minimize the regret, which is the amount by which the total loss incurred exceeds the total loss of the best decision in hindsight. We study the case where decisions and outcomes lie in a convex subset of a Hilbert space, and loss is squared distance. When the set is the simplex, this is the 'Brier game,' studied for the calibration of sequential probability forecasts; when it is the Euclidean ball, the game is related to sequential Gaussian density estimation. We show that the value of the game depends only on the radius of the smallest ball that contains the convex subset, and that the minimax optimal strategy is a simple shrinkage strategy that can be efficiently computed, given the center of the smallest ball.

Joint work with Wouter Koolen (UC Berkeley and Queensland University of Technology) and Alan Malek (UC Berkeley).

C3 - December 18, 17:00 – 17:30

Simultaneous Model Selection and Learning through Parameter-free Stochastic Gradient Descent

Francesco Orabona
Yahoo! Labs NY, USA
francesco@orabona.com

Stochastic gradient descent algorithms for training linear and kernel predictors are gaining more and more importance, thanks to their scalability. While various methods have been proposed to speed up their convergence, the issue of the model selection phase has often been ignored in the literature. In fact, in theoretical works most of the time unrealistic assumptions are made, for example, on the prior knowledge of the norm of the optimal solution. Hence, costly validation methods remain the only viable approach in practical applications. In this talk, we show how a family of kernel-based stochastic gradient descent algorithms can perform model selection while training, with no parameters to tune, nor any form of cross-validation, and only one pass over the data. These algorithms are based on recent advancements in online learning theory in unconstrained settings. Optimal rates of convergence will be shown under standard smoothness assumptions on the target function, as well as empirical results.

C3 - December 18, 17:30 – 18:00

Trivial Pursuit: A Shallow Learning Retrospective

Benjamin Recht
University of California, Berkeley, USA
brecht@eecs.berkeley.edu

In this talk, I review the history of random features in machine learning over the last ten years. Presenting both my work on the subject and a survey of the related literature, I will show how shallow banks of random features are able to match the performance of complicated learned or engineered features on a variety of difficult learning tasks. I will then describe the present challenges in random feature design and our progress towards making these features scalable, parsimonious, and interpretable.

C3 - December 18, 18:00 – 18:30

The Wasserstein Barycenter Problem: Formulation, Computation and Applications

Marco Cuturi
Kyoto University, Japan
mcuturi@i.kyoto-u.ac.jp

How can we define the average of a set of probability measures? This question is important because (1) averaging ranks among the most elementary operations used in statistics to summarize data (2) probability measures are ubiquitous in machine learning, where they are used to represent datasets, generative models or complex objects (an image as a bag-of-features, a text as a bag-of-words).

I will present in this talk a possible answer to this question grounded on the optimal transport (a.k.a. Wasserstein/ earth mover's) geometry. The problem I will describe, known as the Wasserstein barycenter problem, tries to find, given a set of probability measures of interest, the probability measure that minimizes the sum of all its Wasserstein-distances to those probability measures. After providing a few self-contained reminders on optimal transport in the first part of the talk, I will illustrate using toy data that Wasserstein barycenters have several intuitive and appealing properties. I will then show that in its original form theWasserstein barycenter problem is intractable, but that it can be solved approximately, very efficiently, and to arbitrary precision in practice by regularizing it with an entropic term. I will provide details of very recent algorithmic advances in this nascent field followed by an application to the visualization of datasets of brain activation maps.

C3 - December 19, 14:30 – 15:00

Learning a Hidden Basis through Imperfect Measurements: Why and How

Misha Belkin
Ohio State University, USA
mbelkin@cse.ohio-state.edu

In this talk I will describe a general framework of inferring a hidden basis from imperfect measurements. I will show that a number of problems from the classical eigendecompositions of symmetric matrices to such topics of recent interest as multiclass spectral clustering, Independent Component Analysis and Gaussian mixture learning can be viewed as examples of hidden basis learning.

I will then describe algorithms for basis recovery and provide theoretical guarantees in terms of computational complexity and perturbation size. The proposed algorithms are based on what may be called "gradient iteration" and are simple to describe and to implement. They can be viewed as generalizations of both the classical power method for recovering eigenvectors of symmetric matrices as well as the recent work on power methods for tensors. Unlike these methods, our analysis is based not on tensorial properties, but on certain "hidden convexity" of constrast functions.

Joint work with L. Rademacher (Ohio State University) and J. Voss (Ohio State University).

C3 - December 19, 15:00 – 15:30

Tensor decomposition, convex optimization, and multitask learning

Ryota Tomioka
Toyota Technological Institute at Chicago, USA
tomioka@ttic.edu

Tensor factorization, or multilinear modelling, has received much attention recently. Compared to its two-dimensional counterpart, matrix factorization, many properties related to tensors, for example, the rank, are known to be hard to compute. Recently new approaches based on convex relaxation of tensor (multilinear-)rank have emerged. Although, these new methods come with worst case performance guarantees, they tend to be less efficient than previously known greedy algorithms in practice. I will overview and discuss the possibility and limitation of these approaches from the perspective of computation-statistics trade-off. Furthermore, I will present a recent application of the above idea to multi-task learning.

Joint work with Kishan Wimalawarne (Tokyo Institute of Technology,), Taiji Suzuki (Tokyo Institute of Technology), Kohei Hayashi (National Institute of Informatics, Tokyo), Hisashi Kashima (Kyoto University) and Masashi Sugiyama (University of Tokyo).

C3 - December 19, 15:30 – 16:00

Democratic Learning: Learning to Represent Data for Everybody

Guillermo Sapiro
Duke University, USA
guillermo.sapiro@duke.edu

In this talk I will describe a simple framework for learning data transforms that are computationally free and help diverse classification and clustering algorithms. When incorporated into standard techniques such as subspace clustering, random forests, and hashing codes, we obtain one to two orders of magnitude improvement at virtually no cost. I will present both the underlying concepts and applications ranging from scene recognition to image classification to 3D object analysis.

Joint work with Qiang Qiu (Duke University) and Alex Bronstein (Tel Aviv University).

C3 - December 19, 16:00 – 16:30

Stability and statistical properties of topological information inferred from data

Frederic Chazal
INRIA Saclay, France
frederic.chazal@inria.fr

Computational topology has recently seen an important development toward data analysis, giving birth to Topological Data Analysis. Persistent homology appears as a fundamental tool in this field. It is usually computed from filtrations built on top of data sets sampled from some unknown (metric) space, providing "topological signatures" revealing the structure of the underlying space. In this talk we will present a few stability and statistical properties of persistence diagrams that allow to efficiently compute relevant topological signatures that are robust to the presence of outliers in the data.

C3 - December 19, 17:00 – 17:30

Algebraic Combinatorial Single-Entry Low-Rank Matrix Completion

Franz Kiraly
University College London, UK
f.kiraly@ucl.ac.uk

In recent years, the low-rank matrix completion model has enjoyed quite some success for recommendation and prediction learning. Many standard algorithms in the field are designed for completing the whole matrix (or derivates) - which also means that achieving scalability on huge data sets is a challenging task.

The algebraic combinatorial approach aims for scalability in a canonical and non-heuristic way: instead of considering "global" properties of the matrix - e.g. low nuclear norm, sparse factorization - the underlying theory describes low-rankness "locally", in terms of algebraic relations between small numbers of entries and combinatorial properties of the observation pattern, both closely interrelated. Algorithmically, this allows to obtain estimates for single entries and error bounds while looking only at a small part of the observation data in the "combinatorial neighbourhood", described by the bipartite graph of observations.

The algebraic combinatorial approach therefore allows, for the first time, a systematic treatment of single-entry estimates including single-entry error bounds, and it yields, for the first time, a closed approach to the low-rank model that is intrinsically local.

In the talk, I will give a brief introduction to the matrix completion problem and its algebraic combinatorial formulation; I will demonstrate how this allows to derive simple reconstruction algorithms, and review some recent empirical results.

Joint work with Duncan Blythe (TU Berlin), Louis Theran (Aalto University, Helsinki) and Ryota Tomioka (TTI Chicago).

C3 - December 19, 17:30 – 18:00

Neurally Plausible Algorithms Find Globally Optimal Sparse Codes

Ankur Moitra
Massachusetts Institute of Technology, USA
moitra@mit.edu

We prove that neurally plausible algorithms — including the classic one identified by Olshausen and Field — can efficiently find a basis that enables sparse representation of a dataset, a foundational problem in neural computation and machine learning. This problem involves non-convex optimization. However, under plausible conditions where the global optimum is unique, we show that the algorithms converge rapidly and with near-optimal sample complexity, to the global optimum. This suggests that non-convexity need not be a hurdle to a rigorous mathematical and algorithmic theory of neural computation.

Joint work with Sanjeev Arora (Princeton University), Rong Ge (Microsoft Research) and Tengyu Ma (Princeton University).

C3 - December 20, 14:35 – 15:25

Kernel-based learning methods

Ingo Steinwart
University of Stuttgart, Germany
ingo.steinwart@mathematik.uni-stuttgart.de

The last decade has witnessed an explosion of data collected from various sources. Since in many cases these sources do not obey the assumptions of classical statistical approaches, new automated methods for interpreting such data have been developed in the machine learning community. Statistical learning theory tries to understand the statistical principles and mechanisms these methods are based on.

This talk begins by introducing some central questions considered in statistical learning. Then various theoretical aspects of a popular class of learning algorithms, which include support vector machines, are discussed. In particular, I will describe how classical concepts from approximation theory such as interpolation spaces and entropy numbers are used in the analysis of these methods. The last part of the talk considers more practical aspects including the choice of the involved loss function and some implementation strategies. In addition, I will present a data splitting strategy that enjoys the same theoretical guarantees as the standard approach but reduces the training time significantly.

C3 - December 20, 15:30 – 16:00

Sparse Estimation with Strongly Correlated Variables

Robert Nowak
University of Wisconsin-Madison, USA
nowak@ece.wisc.edu

This talk considers ordered weighted L1 (OWL) norm regularization for sparse estimation problems with strongly correlated variables. We show that OWL norm regularization automatically clusters strongly correlated variables, in the sense that the coefficients associated with such variables have equal estimated values. Furthermore, we characterize the statistical performance of OWL norm regularization for generative models in which certain clusters of regression variables are strongly (even perfectly) correlated, but variables in different clusters are uncorrelated. We show that if the true p-dimensional signal generating the data involves only s of the clusters, then O(s log p) samples suffice to accurately estimate the signal, regardless of the number of coefficients within the clusters. The estimation of s-sparse signals with completely independent variables requires just as many measurements. In other words, using the OWL we pay no price (in terms of the number of measurements) for the presence of strongly correlated variables.

C3 - December 20, 16:00 – 16:30

Learning Theory and Adaptive Partitioning in High Dimensions

Peter Binev
University of South Carolina, USA
binev@mailbox.sc.edu

Adaptive partitioning has been one of the methods of choice for several problems from nonlinear approximation theory. A typical challenge in applying this approach to learning theory is the increased complexity of the high-dimensional realization of the adaptive algorithms. We discuss paradigms like sparse occupancy and decorated trees that are designed to alleviate the difficulties related to high dimensions and tuned to certain learning theory setups.

C3 - December 20, 17:00 – 17:30

Learning of invariant representations in visual cortex: i-theory

Tomaso Poggio
Massachusetts Institute of Technology, USA
tp@ai.mit.edu

I-theory is developed starting from the hypothesis that invariant representations of images are the main computational goal of the ventral stream in visual cortex. We prove that invariant representations lead to lower sample complexity in image recognition. We propose a biologically plausible simple-complex cells module (HW module) for computing components of an invariant signature. For transformations that have the structure of a locally compact group we prove invariance and selectivity, showing how a hierarchical architecture of HW modules can learn in an unsupervised way to be automatically invariant to transformations of a new object, achieving the goal of recognition with very few labeled examples. I-theory makes specific predictions about the architecture of the ventral stream, including the dependence on eccentricity of the magnification factor in various areas, and on the tuning properties of its neurons from early generic, Gabor-like tuning to class-specific tuning in AIT.

This approach is an example of what could be the next phase in the theory of learning: how to learn in an unsupervised way good representations that allow a supervised classifier to learn from very few labeled examples, similar to how children learn.

C3 - December 20, 17:30 – 18:00

Iterative Regularization for Computational Learning

Lorenzo Rosasco
Universita' di Genova , Italy
lrosasco@mit.edu

Iterative regularization approaches to ill-posed inverse problems are known to provide a viable alternative to Tikhonov regularization, especially in large scale problems. Supervised learning can be seen as an inverse problem under a suitable stochastic data model. In this context, iterative regularization is particularly suited since statistical and computational aspects are tackled at once, a key property when dealing with large data-sets. In this talk we will discuss old and new results on learning with iterative regularization and connect them with recent results in online learning.

Workshop C4
Numerical Linear Algebra

Organizers: Daniel Kressner – Olga Holtz – Alan Edelman

C4 - December 18, 14:35 – 15:25

A practical framework for infinite-dimensional linear algebra

Sheehan Olver
The University of Sydney, Australia
Sheehan.Olver@sydney.edu.au

We describe a framework for solving a broad class of infinite dimensional linear equations, consisting of almost banded operators, which can be used to resepresent linear ordinary differential equations with general boundary conditions. The framework contains a data structure on which row operations can be performed, allowing for the solution of linear equations by the adaptive QR approach. The algorithm achieves $O(n)$ complexity, where n is the number of degrees of freedom required to achieve a desired accuracy, which is determined adaptively. In addition, special tensor product equations, such as partial differential equations on rectangles, can be solved by truncating the operator along one dimension and using a generalized Schur decomposition. The framework is implemented in the ApproxFun.jl package written in the Julia programming language.

C4 - December 18, 15:30 – 16:00

Applications of the GSVD

Yuyang Wang
Amazon, USA
wangyuyang1028@gmail.com

We describe the use of the generalized singular value decomposition in numerical linear algebra and random matrix theory. The gsvd is well known in numerical linear algebra but not as well known in other circles. We describe applications.

Joint work with Alan Edelman (MIT, USA).

C4 - December 18, 16:00 – 16:30

Julia: A Fresh Approach to Technical Computing

Alan Edelman
MIT, USA
edelman@mit.edu

I will introduce new users to Julia, and tell some of the history of the project.

C4 - December 18, 17:05 – 17:55

Factoring arbitrary matrices into products of structured matrices

Lek-Heng Lim
University of Chicago, USA
lekheng@galton.uchicago.edu

We show that every $n \times n$ matrix can be decomposed into (i) a product of $n/2$ Toeplitz matrices, (ii) a product of $n/2$ Hankel matrices, (iii) a product of $4n$ Vandermonde matrices, (iv) a product of $16n$ bidiagonal matrices, or (v) a product of n^2 companion matrices. We will see that such decompositions do not in general hold with other types of structured matrix factors (e.g. circulant, symmetric Toeplitz, persymmetric Hankel, etc).

Joint work with Ke Ye (University of Chicago, USA).

C4 - December 18, 18:00 – 18:30

Applications of the Cauchon Algorithm

Juergen Garloff
HTWG Konstanz, Gemany
garloff@htwg-konstanz.de

During the last years, the Cauchon algorithm (also called deleting derivation algorithm [5] and Cauchon reduction algorithm [6]) has been applied in algebra, e.g., to tours invariant prime ideals in the quantum matrices. In [5] and [6] this algorithm was employed for the recognition of totally nonnegative cells. A real matrix is called totally nonnegative if all its minors are nonnegative. Such matrices arise in a variety of ways in mathematics and its applications, e.g., in differential and integral equations, numerical mathematics, combinatorics, statistics, and computer aided geometric design. For background information we refer to the recently published monographs [3], [7]. In [2] we apply the Cauchon algorithm [5], [6] for a proof of a conjecture posed by the speaker in 1982 [4], see also [3, Section 3.2] and [7, Section 3.2]. This conjecture originated in the interpolation of interval-valued data by using B-splines. It concerns the checkerboard ordering which is obtained from the usual entry-wise ordering in the set of the square real matrices of fixed order by reversing the inequality sign for each entry in a checkerboard fashion. The conjecture concerns the interval property of the nonsingular totally nonnegative matrices, i.e., if the two bound matrices of an interval with respect to this ordering are nonsingular and totally nonnegative then all matrices lying between the two bound matrices are nonsingular and totally nonnegative, too.

In our talk we also brie y report on some very recent applications of the Cauchon algorithm, viz.

• new determinantal tests for testing a given matrix for total nonnegativity and related properties which require much fewer minors to be checked as the tests known so far,

• finding for each entry of a nonsingular totally nonnegative matrix the largest amount by which this entry can be perturbed without losing the property of total nonnegativity,

• identifying other subclasses exhibiting the interval property of the sign regular matrices, i.e., of matrices with the property that all their minors offixed order have one specfied sign or are allowed also to vanish.

References and Literature for Further Reading:

[1] M. Adm and J. Garloff, Invariance of total nonnegativity of a tridiagonal matrix under element-wise perturbation, Oper. Matrices 8 (1) (2014) 129-137.

[2] M. Adm and J. Garloff, Intervals of totally nonnegative matrices, Linear Algebra Appl., 439 (2013) 3796-3806.

[3] S. M. Fallat and C. R. Johnson, Totally Nonnegative Matrices, Princeton Series in Applied Mathematics, Princeton University Press, Princeton and Oxford, 2011.

[4] J. Garloff, Criteria for sign regularity of sets of matrices, Linear Algebra Appl., 44 (1982) 153-160.

[5] K. R. Goodearl, S. Launois and T. H. Lenagan, Totally nonnegative cells and matrix Poisson varieties, Adv. Math., 226 (2011) 779-826.

[6] S. Launois and T. H. Lenagan, Efficient recognition of totally nonnegative matrix cells, Found. Comput. Mat., 14 (2) (2014) 371-387.

[7] A. Pinkus, Totally Positive Matrices, Cambridge Tracts in Mathematics 181, Cambridge Univ. Press, Cambridge, UK, 2010.

Joint work with Mohammad Adm (University of Konstanz, Germany).

C4 - December 19, 14:30 – 15:00

Complexity of homotopy methods for the eigenvalue problem I

Felipe Cucker
City University of Hong Kong, Hong Kong
macucker@cityu.edu.hk

We describe the shortcomings of the current algorithmic solutions for the eigenvalue problem and the basic ingredients for a homotopy method for this problem.

C4 - December 19, 15:05 – 15:55

Complexity of homotopy methods for the eigenvalue problem II

Diego Armentano
Universidad de la República, Uruguay
diego@cmat.edu.uy

We describe possible choices for the initial triple (matrix, eigenvalue, eigenvector) for the homotopy and their corresponding average complexity analyses in the Hermitian and general case.

C4 - December 19, 16:00 – 16:30

Low-rank tensor completion by Riemannian optimization

Daniel Kressner
EPFL, Switzerland
daniel.kressner@epfl.ch

In tensor completion, the goal is to fill in missing entries of a partially known tensor under low-rank constraints. We consider low-rank formats that form Riemannian manifolds, such as the Tucker or the tensor train (TT) format. This allows for the application of Riemannian optimization techniques for solving the tensor completion problem. In particular, the nonlinear CG algorithm can be implemented such that it scales linearly in the size of the tensor. We illustrate the use of this algorithm for approximating multivariate functions as well as for parameter-dependent and stochastic partial differential equations.

C4 - December 19, 17:05 – 17:55

On low-rank approximability of solutions to Kronecker-structured operator equations

André Uschmajew
University of Bonn, Germany
uschmajew@ins.uni-bonn.de

Let A and B be invertible, and let C have low rank. Then it is trivial that the rank of the solution to the matrix equation $AXB^T = C$ is not larger than the rank of C. Formally speaking, this matrix equation admits a low-rank solution. There is some numerical evidence that the solution of frequently occurring linear equations like $A_1XB_1^T + \cdots + A_RXB_R^T = C$, where the number of terms is still considerably smaller than the matrix dimension (which might be infinite in an Hilbert space setting), has good low-rank approximability, that is, fast decaying singular values. Exponential decay, as sometimes observed in practice, seems very challenging to prove. It is, however, rather easy to obtain a nontrivial algebraic decay rate by relating the convergence speed of some polynomial approximation to the Kronecker rank of the corresponding operator polynomial. For an eigenvalue problem $AXB^T = \lambda X$ a similar argumentation is possible, although with considerable restrictions. This simple method of estimating the low-rank approximation error for matrices has important consequences for the low-rank approximability of solutions to Kronecker-structured operator equations in higher-order tensor product spaces, as it provides estimates on the singular value decay of certain matrix unfoldings of the solution tensor, which in turn govern the error in higher-order SVD truncations. The talk is based on joint-work with Daniel Kressner.

C4 - December 19, 18:00 – 18:30

Approximation with cross-kernel matrices, and Ideal PCA

Franz Király
University College London, United Kingdom
f.kiraly@ucl.ac.uk

We describe how cross-kernel matrices, that is, kernel matrices between the data and a custom chosen set of 'feature spanning points' can be used for learning. At the core is a Nyström-type relation which allows to numerically approximate kernel matrices in terms of cross-kernel matrices. We demonstrate how cross-kernel matrices (A) provide a universal speed-up of kernel learning algorithms through replacing kernel matrices with cross-kernel matrices that scale linearly in the data, and (B) allow learning of features which approximately vanish on the data. We further give an algorithm, called ideal principal component analysis (IPCA), which we derive as the cross-kernel variant of kernel PCA, serving as an exemplification of both the new features and the speed-up to linear runtime. We conclude with some open questions regarding the approximating properties of cross-kernel matrices.

Joint work with Louis Theran, Martin Kreuzer.

Workshop C5
Special Functions and Orthogonal Polynomials

Organizers: Peter Clarkson – Kerstin Jordaan – Francisco (Paco) Marcellán

---

C5 - December 18, 14:30 – 15:00

Extending Askey tableau by the inclusion of Krall and exceptional polynomials

Antonio J. Durán
Universidad de Sevilla, Spain
duran@us.es

Krall and exceptional polynomials are two of the more important extensions of the classical families of Hermite, Laguerre and Jacobi. On the one hand, Krall or bispectral polynomials are orthogonal polynomials which are also eigenfunctions of a differential operator of order bigger than two (and polynomial coefficients). The first examples were introduced by H. Krall in 1940, and since the eighties a lot of effort has been devoted to this issue (with contributions by L.L. Littlejohn, A.M. Krall, J. and R. Koekoek. A. Grünbaum and L. Haine (and collaborators), K.H. Kwon (and collaborators), A. Zhedanov, P. Iliev, and many others). On the other hand, exceptional polynomials are orthogonal polynomials which are also eigenfunctions of a second order differential operator, but they differ from the classical polynomials in that their degree sequence contains a finite number of gaps, and hence the differential operator can have rational coefficients. In mathematical physics, these functions allow to write exact solutions to rational extensions of classical quantum potentials. Exceptional polynomials appeared some seven years ago, but there has been a remarkable activity around them mainly by theoretical physicists (with contributions by D. Gómez-Ullate, N. Kamran and R. Milson, Y. Grandati, C. Quesne, S. Odake and R. Sasaki, and many others). Taking into account these definitions, it is scarcely surprising that no connection has been found between Krall and exceptional polynomials. However, if one considers difference operators instead of differential ones (that is, the discrete level of Askey tableau), something very exciting happens: Duality (i.e., swapping the variable with the index) interchanges Krall discrete and exceptional discrete polynomials. This unexpected connection of Krall discrete and exceptional polynomials allows a nice and important extension of Askey tableu. Also, this worthy fact can be used to solve some of the most interesting questions concerning exceptional polynomials; for instance, to find necessary and sufficient conditions such that the associated second order differential operators do not have any singularity in their domain.

C5 - December 18, 15:00 – 15:30

Orthogonal and para-orthogonal polynomials on the unit circle

A. Sri Ranga
UNESP - Universidade Etadual Paulista, Brazil
ranga@ibilce.unesp.br

When a nontrivial measure μ on the unit circle satisfies the symmetry $d\mu(e^{i(2\pi-\theta)}) = -d\mu(e^{i\theta})$ then the associated orthogonal polynomials on the unit circle, say S_n, are all real. In this case, in [3], Delsarte and Genin have shown that the two sequences of para-orthogonal polynomials $\{zS_n(z) + S_n^*(z)\}$ and

$\{zS_n(z) - S_n^*(z)\}$ satisfy three term recurrence formulas and have explored some further consequences of these sequences of polynomials such as their connections to sequences of orthogonal polynomials on the interval $[-1, 1]$. Even though results presented in Delsare and Genin [4] extend these partly to include any nontrivial measures on the unit circle, only recently, in [2] (and also [1]), the extension associated with the para-orthogonals polynomials $zS_n(z) - S_n^*(z)$ was studied extensively. The results given in [2], especially from the point of view of three term recurrence, provide also as a nice application a characterization for any pure points in the measure. The main objective of the present contribution is to provide some recent developments concerning the extension for the para-orthogonals polynomials $zS_n(z) + S_n^*(z)$ to cover all nontrivial measures on the unit circle.

References

[1] K. Castillo, M. S. Costa, A. Sri Ranga and D. O. Veronese, A Favard type theorem for orthogonal polynomials on the unit circle from a three term recurrence formula, J. Approx. Theory, 184 (2014), 146-162.

[2] M.S. Costa, H.M. felix and A. Sri Ranga, Orthogonal polynomials on the unit circle and chain sequences, J. Approx. Theory, 173 (2013), 14-32.

[3] P. Delsarte and Y. Genin, The split Levinson algorithm, IEEE Trans. Acoust. Speech Signal Process, 34 (1986), 470-478.

[4] P. Delsarte and Y. Genin, The tridiagonal approach to Szegő's orthogonal polynomials, Toeplitz linear system, and related interpolation problems, SIAM J. Math. Anal., 19 (1988), 718-735.

Joint work with Cleonice F. Bracciali (Universidade Estadual Paulista, Brazil) and A. Swaminathan (IIT Roorkee, India).

C5 - December 18, 15:30 – 16:00

On orthogonal polynomials associated with perturbations of Hankel matrices

Luis E. Garza
Universidad de Colima, México
garzaleg@gmail.com

In this contribution, we study algebraic properties of orthogonal polynomials associated with perturbations of Hankel matrices. In particular, we give explicit expressions for such polynomials, and obtain some properties associated with their zeros.

Joint work with K. Castillo (Universidad Estadual Paulista, Brazil), D. Dimitrov (Universidad Estadual Paulista, Brazil) and F. Rafaeli (Universidad Estadual Paulista, Brazil).

C5 - December 18, 16:00 – 16:30

Kissing polynomials

Arieh Iserles
University of Cambridge, United Kingdom
ai@damtp.cam.ac.uk

Our point of departure are Gauss-like methods for highly oscillatory quadrature of integrals $\int_{-1}^{1} f(x)e^{\mathrm{i}\omega x}\mathrm{d}x$, $\omega \gg 1$, employing orthogonal polynomials with a complex-valued weight function. Such polynomials pose fascinating questions of existence and their zeros, plotted in the complex plane as a function of the underlying frequency ω, exhibit an intriguing pattern of "kissing". All this will be analysed and elucidated in the talk. Our main tool is asymptotic analysis for $\omega \gg 1$ of Hankel determinants, represented by multivariate highly oscillatory integrals, and their generalisations.

Joint work with Andreas Asheim (NTNU Trondheim), Alfredo Deaño (KU Leuven) and Daan Huybrechs (KU Leuven).

C5 - December 18, 17:00 – 17:30

Orthogonal polynomials and integral transforms

Ana F. Loureiro
University of Kent, U.K.
A.Loureiro@kent.ac.uk

In this talk I will explain how operating with certain integral transforms over polynomial sequences is a useful tool to obtain and deduce properties of one sequence based on the other. A special attention will be given to certain d-orthogonal polynomial sequences, which basically are polynomial sequences satisfying a recurrence relation of order $d + 1$. When $d = 1$, we recover the orthogonal case. Examples of some known polynomial sequences with a plank of applications will be used to illustrate the usefulness of the technique. Among the targeted sequences, some semiclassical polynomials will arise.

C5 - December 18, 17:30 – 18:00

Explicit formulas for OPUC and para-orthogonal polynomials for measures which are modifications of Lebesque measure

Cleonice Bracciali
UNESP - Universidade Estadual Paulista, Brazil
cleonice@ibilce.unesp.br

We consider nontrivial probability measures, obtained as simple modifications of the Lebesgue measure, which include mass points at $z = 1$ and $z = i$. The orthogonal polynomials on the unit circle (OPUC), the para-orthogonal polynomials and Toeplitz matrices associated with these measures are presented, through explicit formulas for the Verblunsky coefficients.

Joint work with A. Sri Ranga (UNESP - Universidade Estadual Paulista, Brazil).

C5 - December 18, 18:00 – 18:30

Spectral Orthogonal Polynomals and Differential Galois Theory

Primitivo Acosta-Humánez
Universidad del Atlántico, Colombia
primi@intelectual.co

In this talk we present a Differential Galois Theory approach to study the Schrödinger equation with polynomial potentials. We obtain the algebraic spectrum, values of energy in which the Schrodinger

equation is integrable in Galoisian sense, of harmonic and anharmonic oscillators. In particular, for the sextic anharmonic oscillator we recover the classical Bender - Dunne polynomials as spectral orthogonal polynomials, that is, the algebraic spectrum of the Schrödinger equation with polynomial potential of degree six which corresponds to the zeroes of the Bender-Dunne spectral polynomials. Generalizations of this approach, such as decatic potentials, are also presented.

Joint work with Henock Venegas (Universidad del Atlántico, Colombia).

C5 - December 19, 14:30 – 15:20

Semi-classical orthogonal polynomials and the Painlevé equations

Peter Clarkson
University of Kent, UK
P.A.Clarkson@kent.ac.za

In this talk I shall discuss the relationship between the Painlevé equations and orthogonal polynomials with respect to semi-classical weights. It is well-known that orthogonal polynomials satisfy a three-term recurrence relation. I will show that for some semi-classical weights, the coefficients in the recurrence relation can be expressed in terms of Wronskians that arise in the description of special function solutions of a Painlevé equation. The orthogonal polynomials discussed will include semi-classical Laguerre and Hermite weights, orthogonal polynomials with discontinuous weights and semi-classical generalizations of the Charlier and Meixner polynomials, which are discrete orthogonal polynomials.

Joint work with Kerstin Jordaan (University of Pretoria, South Africa) and J G Smith (University of Kent, United Kingdom).

C5 - December 19, 15:30 – 16:00

Bispectrality, the Darboux process and time-band limiting

Mirta María Castro Smirnova
University of Seville, Spain
mirta@us.es

We consider a few examples illustrating connections among the notions in the title.

Joint work with Alberto Grünbaum (University of California, Berkeley).

C5 - December 19, 16:00 – 16:30

Orthogonal Rational Functions and non-stationary stochastic processes

Laurent Baratchart
INRIA Sophia-Antipolis, France
Laurent.Baratchart@inria.fr

We present a generalization of the Szegö theory of orthogonal polynomials on the unit circle to orthogonal rational functions. Just like the Kolmogorov-Krein-Szegö theorem may be interpreted as an asymptotic estimate of the prediction error for stationary stochastic processes, the present theory yields an asymptotic estimate of the prediction error for certain, possibly non-stationary, stochastic processes. The latter admit a spectral calculus where the time-shift corresponds to multiplication by elementary Blaschke products of degree 1 (that reduce to multiplication by the independent variable in the stationnary case). When the poles of the best predictor tend to a point on the unit circle where the spectral density is non-zero, the prediction error goes to zero, that is, the process is asymptotically deterministic.

Joint work with Leonid Golinskii (Mathematics Division, Institute for Low Temperature Physics and Engineering, Kharkov, Ukraine) and Stanislas Kupin (Mathematics Department, University Bordeaux I, France).

C5 - December 19, 17:00 – 17:30

Multiple orthogonal polynomials associated with an exponential cubic weight

Walter Van Assche
KU Leuven, Belgium
walter@wis.kuleuven.be

We consider multiple orthogonal polynomials associated with the exponential cubic weight e^{-x^3} over two contours in the complex plane. We give the basic properties of these polynomials, including the Rodrigues formula and nearest-neighbor recurrence relations. It turns out that the recurrence coefficients are related to a discrete Painlevé equation. The asymptotics of the recurrence coefficients, the ratio of the diagonal multiple orthogonal polynomials and the (scaled) zeros of these polynomials are also investigated.

Joint work with Galina Flipuk (Univerity of Warsaw, Poland) and Lun Zhang (Fudan University, Shanghai, China).

C5 - December 19, 17:30 – 18:00

Quadratic Algebras of Orthogonal Polynomials

Sarah Post
University of Hawaii, USA
spost@hawaii.edu

In this talk, we will discuss the connection between superintegrable systems and classical systems of orthogonal polynomials in particular in the expansion coefficients between separable coordinate systems, related to representations of the (quadratic) symmetry algebras. This connection allows us to extend the Askey scheme of classical orthogonal polynomials and the limiting processes within the scheme. In particular, for superintegrable systems in 3D, the polynomial representations of quadratic algebras are given in terms of two-variable polynomials and the two-variable analog of the Askey scheme, including the quadratic Racah algebra, will be discussed.

C5 - December 19, 18:00 – 18:30

G-function of Meijer and generalized hypergeometric function: interplay of new facts

Dmitrii Karp
Far Eastern Federal University, Russian Federation
dimkrp@gmail.com

We discuss various new properties of Meijer's G-function $G^{p,0}_{p,p}$, including integral and functional equations, nonnegativity conditions and number of zeros, convergence of measures with G-function density and regularization of integrals containing G-function. Some of these properties are then applied to derive new representations for generalized hypergeometric functions and establish some new and old facts about them. In particular, we prove log-convexity in upper parameters, demonstrate monotonicity of certain ratios and find new proofs for Luke's inequalities permitting their extension to wider parameter ranges. We further find an upper bound for the Gauss type generalized hypergeometric function not previously contained in the literature. Finally, using a different approach we give new two-sided bounds for the Bessel type generalized hypergeometric function of nonnegative argument.

The talk is based on joint work with J.L.Lopez and E.G.Prilepkina. The research has been supported by the Ministry of Education and Science of the Russian Federation under project 1398.2014.

C5 - December 20, 14:30 – 15:00

Applications of infinite matrices in the theories of orthogonal polynomials and operational calculus

Luis Verde-Star
Universidad Autonoma Metropolitana, Mexico
verde@xanum.uam.mx

We use some algebras of infinite matrices $[a_{j,k}]$, where the indexes run over all the integers, to study sequences of polynomials and formal power series and also for the construction of a general operational calculus that can be used to solve linear functional equations of several types.

We consider infinite matrices of the form $\sum_k D_k X^k$, where the D_k are diagonal matrices, X is a shift, and $D_k \neq 0$ for only a finite number of negative values of k. Several basic properties and characterizations of orthogonal polynomial sequences are expressed in terms of infinite matrices.

This work extends some of the results obtained in our previous papers

L. Verde-Star, Characterization and construction of classical orthogonal polynomials using a matrix approach, Linear Algebra Appl. 438 (2013) 3635–3648.

G. Bengochea, L. Verde-Star, Linear algebraic foundations of the operational calculi, Adv. Appl. Math. 47 (2011) 330–351.

C5 - December 20, 15:00 – 15:30

Branching formula for Macdonald-Koornwinder polynomials

Jan Felipe van Diejen
Universidad de Talca, Chile
diejen@inst-mat.utalca.cl

The six-parameter Macdonald-Koornwinder polynomials with hyperoctahedral symmetry form a multivariate generalization of the well-known Askey-Wilson polynomials. We present a branching formula that expands the Macdonald-Koornwinder polynomials in n+1 variables in terms of the n-variable polynomials. This formula allows one to construct the Macdonald-Koornwinder polynomials explicitly by induction in the number of variables.

Joint work with Erdal Emsiz (Pontificia Universidad Católica de Chile, Chile).

C5 - December 20, 15:30 – 16:00

Hermite-Padé Approximants for Angelesco Systems

Maxim Yattselev
Indiana University-Purdue University Indianapolis, USA
maxyatts@iupui.edu

I will discuss asymptotics of type II Hermite-Padé approximants (a vector of rational functions with a common denominator) for a vector of Cauchy transforms of analytic densities along any ray of multi-indices. It is assumed that the densities are supported on mutually disjoint intervals (an Angelesco system with complex weights). The formulae of strong asymptotics will be presented, the relevant Riemann surfaces discussed as well as some details of the local Riemann-Hilbert analysis.

C5 - December 20, 16:00 – 16:30

A q-generalization of the Bannai–Ito polynomials and the quantum superalgebra $\mathfrak{osp}_q(1|2)$

Luc Vinet
Université de Montréal, Canada
luc.vinet@umontreal.ca

A q-generalization of the Bannai–Ito polynomials is presented. These basic polynomials are obtained by considering the Racah problem for the quantum superalgebra $\mathfrak{osp}_q(1|2)$. A quantum deformation of the Bannai–Ito algebra is realized by the intermediate Casimir operators entering in the Racah problem. The relation between the q-analogs of the Bannai–Ito polynomials and the q-Racah/Askey-Wilson polynomials is discussed.

Joint work with Vincent X. Genest (CRM) and Alexei Zhedanov (Donetsk).

C5 - December 20, 17:00 – 17:50

Lifting q-difference operators in the Askey scheme of basic hypergeometric polynomials

Natig Atakishiyev
Instituto de Matemáticas, Unidad Cuernavaca, Universidad Nacional Autónoma de México, Mexico
natig@matcuer.unam.mx

We construct an explicit form of a q-difference operator that lifts the continuous q-Hermite polynomials $H_n(x|q)$ of Rogers into the Askey-Wilson polynomials $p_n(x;a,b,c,d|q)$ on the top level in the Askey q-scheme. This operator represents a special convolution-type product of four one-parameter q-difference operators of the form $\epsilon_q(c_q D_q)$, defined as Exton's q-exponential function $\epsilon_q(z)$ in terms of the Askey-Wilson divided q-difference operator D_q. We show also that one can determine another q-difference operator that transforms the orthogonality weight function for the continuous q-Hermite polynomials $H_n(x|q)$ of Rogers up to the weight function, associated with the Askey-Wilson polynomials $p_n(x;a,b,c,d|q)$.

Joint work with Mesuma Atakishiyeva (Facultad de Ciencias, Universidad Autónoma del Estado de Morelos, Cuernavaca, Morelos, México).

C5 - December 20, 18:00 – 18:30

Quasi-orthogonality of some $_pF_q$ hypergeometric polynomials

Kerstin Jordaan
University of Pretoria, South Africa
kjordaan@up.ac.za

We prove the quasi-orthogonality of some general classes of hypergeometric polynomials of the form

$$ {}_pF_q\left(\begin{array}{c} -n, \beta_1 + k, \alpha_3 \ldots, \alpha_p \\ \beta_1, \ldots, \beta_q \end{array}; x\right) = \sum_{m=0}^{n} \frac{(-n)_m(\beta_1 + k)_m(\alpha_3)_m \ldots (\alpha_p)_m}{(\beta_1)_m \ldots (\beta_q)_m} \frac{x^m}{m!} $$

for $k \in \{1, 2, \ldots, n-1\}$ which do not appear in the Askey scheme for hypergeometric orthogonal polynomials. Our results include, as a special case, the order one quasi-orthogonal Sister Celine polynomials

$$ f_n(a, x) = {}_3F_2\left(\begin{array}{c} -n, n+1, a \\ \frac{1}{2}, 1 \end{array}; x\right) = \sum_{m=0}^{n} \frac{(-n)_m(n+1)_m(a)_m}{\left(\frac{1}{2}\right)_m (1)_m} \frac{x^m}{m!} $$

with $a = 2$ and $a = 3/2$ considered by Dickenson in 1961. The location and interlacing of the real zeros of the quasi-orthogonal polynomials are also discussed.

Joint work with Sarah Jane Johnston (University of South Africa, South Africa).

C5 - Poster

The relationship between the fifth Painlevé equation and orthogonal polynomials

Peter Clarkson
University of Kent, United Kingdom
P.A.Clarkson@kent.ac.uk

Here we are concerned with orthogonal polynomials for a deformed Laguerre weight. It is shown that the coefficients of the three-term recurrence relation satisfied by the polynomials can be expressed in terms of Wronskians which involve Kummer functions. These Wronskians are related to special function solutions of the fifth Painlevé equation. Using this relationship we can explicitly write the recurrence relation coefficients in terms of exact solutions of the fifth Painlevé equation.

Joint work with J Smith (University of Kent, UK).

C5 - Poster

Bivariate Lagrange interpolation and quadrature formulas at the node sets of Lissajous curves

Wolfgang Erb
University of Lübeck, Germany
erb@math.uni-luebeck.de

Motivated by an application in Magnetic Particle Imaging, we study bivariate Lagrange interpolation at the node points of Lissajous curves. The resulting theory is a generalization of the polynomial interpolation theory developed for a node set known as Padua points. With appropriately defined polynomial spaces, we show that the node points of Lissajous curves allow unique interpolation. Further, these node sets can be used as sampling points for quadrature rules for integrals with product Chebyshev weights. An explicit formula for the Lagrange polynomials allows to compute the interpolating polynomial with a simple algorithmic scheme. Compared to the already established schemes of the Padua and Xu points, the numerical results for the proposed scheme show similar approximation errors and a similar growth of the Lebesgue constant.

Joint work with Christian Kaethner (University of Lübeck, Germany), Mandy Grüttner (University of Lübeck, Germany) and Thorsten M. Buzug (University of Lübeck, Germany).

C5 - Poster

Completed Stieltjes interlacing of zeros of different orthogonal polynomials

Alta Jooste
University of Pretoria, South Africa
alta.jooste@up.ac.za

Consider the orthogonal sequence $\{p_n\}_{n=0}^{\infty}$. If the polynomials p_n and p_{n-m}, $m = 2, 3, \dots, n-1$, have no common zeros, there exists a real polynomial of degree $m-1$, completely determined by the coefficients in the three term recurrence relation satisfied by the orthogonal sequence $\{p_n\}_{n=0}^{\infty}$, whose real simple zeros provide a set of points that, together with the zeros of p_{n-m}, completely interlace with the zeros of p_n, a property we refer to as completed Stieltjes interlacing. The conditions under which completed Stieltjes interlacing holds between the zeros of polynomials from different orthogonal sequences are studied and this leads to a set of points that can be applied as bounds for the extreme zeros of the polynomials. We apply our results to some discrete orthogonal polynomials and identify new bounds for the extreme zeros of these polynomials.

Joint work with Kerstin Jordaan (University of Pretoria, South Africa).

C5 - Poster

Application of Fourier series for modelling forest fires in the Russian Far East

Elena Oleinik
Far Eastern Federal University, Russia
oleinik.elena@gmail.com

In the report, we discuss the relationship between the frequency of forest fires in the Russian Far East and the bursts of solar activity. We demonstrate the cointegration between the two variables and estimate the parameters of the model involving a Fourier series. The model is then verified against the data and used to forecast the burned area. Such forecasting helps to reduce the environmental and economic damage.

Joint work with Elena Oleinik (Far Eastern Federal University, Russia).

Workshop C6
Stochastic Computation

Organizers: Mike Giles – Arnulf Jentzen – Klaus Ritter

C6 - December 18, 14:30 – 15:00

Multi-Index Monte Carlo: When Sparsity Meets Sampling

Abdul Lateef Haji Ali
King Abdullah University of Science and Technology, Saudi Arabia
abdullateef.hajiali@kaust.edu.sa

We present a novel Multi-Index Monte Carlo (MIMC) method for weak approximation of stochastic models that are described in terms of differential equations either driven by random measures or with random coefficients. The MIMC method is both a stochastic version of the combination technique introduced by Zenger, Griebel and collaborators and an extension of the Multilevel Monte Carlo (MLMC) method first described by Heinrich and Giles. Inspired by Giles's seminal work, instead of using first-order differences as in MLMC, we use in MIMC high-order mixed differences to reduce the variance of the hierarchical differences dramatically. This in turn yields new and improved complexity results, which are natural generalizations of Giles's MLMC analysis, and which increase the domain of problem parameters for which we achieve the optimal complexity, $\mathcal{O}(\mathrm{TOL}^{-2})$. We propose a systematic construction of optimal sets of indices for MIMC based on properly defined profits that in turn depend on the average cost per sample and the corresponding weak error and variance. Under standard assumptions on the convergence rates of the weak error, variance and work per sample, the optimal index set turns out to be of Total Degree (TD) type. In some cases, using optimal index sets, MIMC achieves a better rate for the computational complexity than does the corresponding rate when using Full Tensor sets. Moreover, we present a simple numerical example that illustrates the method and its computational benefits.

Joint work with Fabio Nobile (École Polytechnique Fédérale de Lausanne, Switzerland) and Raul Tempone (King Abdullah University of Science and Technology, Saudi Arabia).

C6 - December 18, 15:00 – 15:30

Multilevel Monte Carlo for the simulation of dilute polymers

Mike Giles
University of Oxford, U.K.
mike.giles@maths.ox.ac.uk

Polymers immersed in a fluid can be modelled as chains connected by finitely-extensible bonds with nonlinear elastic potentials, subject to random forcing. In this work we discuss the simulation of the system of coupled SDEs which results from the modelling, and the use of multilevel Monte Carlo (MLMC) to efficiently estimate expectations arising from the associated equilibrium distribution. One important element is the use of adaptive timestepping which can be incorporated fairly easily into MLMC. Another is the use of a new multilevel coupling idea developed by Glynn and Rhee (2014) for the simulation of equilibrium expectations in the context of contracting Markov Chains.

Joint work with Endre Suli (Oxford), James Whittle (Oxford) and Shenghan Ye (Oxford).

C6 - December 18, 15:35 – 16:25

Multi Level Monte Carlo for Coulomb Collisions in a Plasma

Russel Caflisch
University of California at Los Angeles, United States
caflisch@math.ucla.edu

This talk will describe the application of MLMC to solution of stochastic differential equations (SDEs) describing Coulomb collisions in a plasma. The collisions are between "test particles" and a Maxwellian distribution of "field Particles". The SDEs are three-dimensional and the Levy areas in the Milstein method are not tractable, but acceleration is achieved by reduction in the variance of the simulation with coarsest time step and by the antithetic method or a "Ito linearization method". Preliminary results are presented on MLMC for systems that depend on mean fields.

Joint work with Lee Ricketson (Courant Institute, NYU, USA), Mark Rosin (Pratt Institute, USA), Andris Dimits (Lawrence Livermore National Labs, USA) and Bruce Cohen (Lawrence Livermore National Labs, USA).

C6 - December 18, 17:00 – 17:30

Weak approximation of stochastic differential equations by a multilevel Monte Carlo method using mean square adaptive numerical integration

Håkon Hoel
University of Oslo, Norway
haakonah1@gmail.com

In this talk, we present a multilevel Monte Carlo (MLMC) method for weak approximation of stochastic differential equations (SDE) that uses an a posteriori mean square error (MSE) adaptive Euler–Maruyama step-size control in the numerical integration of SDE realizations. MSE adaptivity is useful for weak approximation MLMC methods since it provides a reliable and efficient way to control the statistical error of the weak approximation MLMC estimator. For a large set of low-regularity weak approximation problems, the adaptive Euler–Maruyama method produces output whose weak error is bounded by $\mathcal{O}(\epsilon)$ at the cost $\mathcal{O}(\epsilon^{-2}|\log(\epsilon)|^4)$. This is a lower asymptotic cost than what can typically be obtained by the uniform time-step Euler-Maruyama MLMC method on the given set of problems. The cost reduction is illustrated in numerical studies.

Joint work with Juho Häppölä (Applied Mathematics and Computational Sciences, KAUST, Thuwal, Saudi Arabia) and Raúl Tempone (Applied Mathematics and Computational Sciences, KAUST, Thuwal, Saudi Arabia).

C6 - December 18, 17:30 – 18:00

Higher order QMC Galerkin Discretization for parametric operator equations

Josef Dick
The University of New South Wales, Australia
josef.dick@unsw.edu.au

We discuss quasi-Monte Carlo methods to approximate the expected values of linear functionals of Petrov-Galerkin discretizations of parametric operator equations which depend on a possibly infinite sequence of parameters. Such problems arise in the numerical solution of differential and integral equations with random field inputs. We analyze the regularity of the solutions with respect to the parameters in terms of the rate of decay of the fluctuations of the input field. If $p \in (0,1]$ denotes the "summability exponent" corresponding to the fluctuations in affine-parametric families of operators, then we prove that deterministic "interlaced polynomial lattice rules" of order $\alpha = \lfloor 1/p \rfloor + 1$ in s dimensions with N points can be constructed using a fast component-by-component algorithm, in $\mathcal{O}(\alpha\, s\, N \log N + \alpha^2\, s^2 N)$ operations, to achieve a convergence rate of $\mathcal{O}(N^{-1/p})$, with the implied constant independent of s. This dimension-independent convergence rate is superior to the rate $\mathcal{O}(N^{-1/p+1/2})$ for $2/3 \le p \le 1$, which was recently established for randomly shifted lattice rules under comparable assumptions. In our analysis we use a non-standard Banach space setting and introduce "smoothness-driven product and order dependent (SPOD)" weights for which we develop a new fast CBC construction.

Joint work with F. Y. Kuo (UNSW), Q. T. Le Gia (UNSW), D. Nuyens (KU Leuven) and Ch. Schwab (ETH Zürich).

C6 - December 18, 18:00 – 18:30

Optimal mesh hierarchies in Multilevel Monte Carlo methods

Erik Schwerin
Kungliga Tekniska Högskolan, KTH, Sweden
schwerin@csc.kth.se

We will discuss how to choose optimal mesh hierarchies in Multilevel Monte Carlo (MLMC) simulations based on uniform discretization methods with general approximation orders and computational costs. We will compare optimized geometric and non-geometric hierarchies and discuss how enforcing some domain constraints on parameters of MLMC hierarchies affects the optimality of these hierarchies. We also discuss the optimal tolerance splitting between the bias and the statistical error contributions and its asymptotic behavior.

Joint work with N. Collier, A. L. Haji-Ali, F. Nobile, and R. Tempone.

C6 - December 19, 14:30 – 15:00

Quadrature for self-affine distributions on $\mathbf{R}^d$

Thomas Mueller-Gronbach
University of Passau, Germany
thomas.mueller-gronbach@uni-passau.de

We study numerical integration of q-times differentiable functions on $\mathbf{R}^d$ for a probability measure that is self-similar with respect to m affine contraction mappings $S_1, \ldots, S_m$ on $\mathbf{R}^d$ and corresponding probability weights $\rho_1, \ldots, \rho_m$. Under mild conditions on the contractions we provide lower bounds for the worst case errors of deterministic as well as randomized algorithms in terms of the worst case (average) number of function evaluations that are used. The matching upper bounds are obtained by composite quadrature rules, which are easy to implement and are based on divide and conquer strategies that are adapted to the structure of the self-similarity. The optimal order of convergence is characterized in terms of the similarity dimension of the contractions.

Joint work with Steffen Dereich (University of Muenster).

C6 - December 19, 15:00 – 15:30

A multilevel stochastic collocation method for PDEs with random inputs

Aretha Teckentrup
University of Warwick, United Kingdom
arethat24@gmail.com

By employing a hierarchy of both spatial approximations and interpolation operators in stochastic parameter space, we develop a multilevel version of stochastic collocation methods for random partial differential equations, leading to a significant reduction in computational cost. We provide a convergence and cost analysis of the new algorithm, and demonstrate the gains possible on a typical random diffusion model problem.

Joint work with Max Gunzburger (Florida State University, USA), Peter Jantsch (University of Tennessee, USA) and Clayton Webster (Oak Ridge National Laboratory, USA).

C6 - December 19, 15:35 – 16:25

Multilevel Monte-Carlo Methods for hyperbolic PDEs with random input data

Christoph Schwab
SAM, ETH Zurich, Switzerland
schwab@math.ethz.ch

We consider random scalar hyperbolic conservation laws (RSCLs) in spatial dimension d with bounded random flux functions which are P-a.s. Lipschitz continuous with respect to the state variable.

There exists a unique random entropy solution (i.e., a strongly measurable mapping from a probability space into $C([0,T];L^1(\mathbb{R}^d)))$ with finite second moments.

We present a convergence analysis of a Multi-Level Monte-Carlo Front-Tracking (MLMCFT) algorithm.

It is based on "pathwise" application of the Front-Tracking Method for deterministic SCLs.

We compare the MLMCFT algorithms to the Multi-Level Monte-Carlo Finite-Volume methods. Due to the absence of a CFL time step restriction in the pathwise front tracking scheme, we can prove favourable complexity estimates: in spatial dimension $d \geq 2$, the mean field of the random entropy solution can be approximated numerically with (up to logarithmic terms) the same complexity as the solution of one instance of the deterministic problem, on the same mesh.

We then present results on large scale simulations of MLMC for wave propagation in heterogeneous media with log-Gaussian random coefficients. Here, conventional explicit timestepping schemes encounter the CFL constraint which, due to the lognormal Gaussian constitutive parameter, is random. A novel probabilistic complexity analysis and and adaptive load balancing algorithm achieve near linear strong scaling on up to 40K processors.

Joint work with Siddhartha Mishra (ETH), Nils Henrik Risebro (Oslo), Jonas Sukys (ETH) and Franziska Weber (Oslo).

C6 - December 19, 17:00 – 17:30

Analytical approximations of BSDEs with non-smooth driver

Emmanuel Gobet
Ecole Polytechnique, France
emmanuel.gobet@polytechnique.edu

We provide and analyse analytical approximations of BSDEs in the limit of small non-linearity and short time to maturity, in the case of non-smooth drivers. We identify the first and the second order approximations within this asymptotics and consider two topical financial applications: the two interest rates problem and the Funding Value Adjustment. In high dimensional diffusion setting, we show how to compute explicitly the first order formula by taking advantage of recent proxy techniques. Numerical tests up to dimension 10 illustrate the efficiency of the numerical schemes. We also show that third order expansions may fail.

Joint work with Stefano Pagliarani (Ecole Polytechnique, France).

C6 - December 19, 17:30 – 18:00

Simulation of forward-reverse stochastic representations for conditional diffusions

Christian Bayer
Weierstrass Institute for Applied Analysis and Stochastics, Germany
christian.bayer@wias-berlin.de

We derive stochastic representations for the finite dimensional distributions of a multidimensional diffusion on a fixed time interval, conditioned on the terminal state. The conditioning can be with respect to a fixed point or more generally with respect to some subset. The representations rely on a reverse process connected with the given (forward) diffusion as introduced in Milstein et al. [Bernoulli 10(2):281–312, 2004] in the context of a forward-reverse transition density estimator. The corresponding Monte Carlo estimators have essentially root-N accuracy, hence they do not suffer from the curse of dimensionality. We provide a detailed convergence analysis and give a numerical example involving the realized variance in a stochastic volatility asset model conditioned on a fixed terminal value of the asset. We show that the algorithm can be applied for inference under incomplete data for the calculation of the expectation step in the EM algorithm. Finally, we give some applications.

Joint work with John Schoenmakers (Weierstrass Institute, Germany).

C6 - December 19, 18:00 – 18:30

Customized fully implementable numerical schemes for FBSDEs

Lukasz Szpruch
University of Edinburgh, UK
l.szpruch@ed.ac.uk

In this talk we introduce a family of explicit numerical approximations for the forward backward stochastic differential equations (FBSDEs). We show that newly developed methodology allows to analyse BSDEs with drivers having polynomial growth and that are also monotone in the state variable. This offers a probabilistic scheme for wide class of reaction-diffusion PDEs. Proposed schemes preserve qualitative

properties of the solutions to the FBSDEs for all ranges of time-steps. We conclude the talk by presenting a new efficient algorithm that allows to approximate conditional expectations in BSDEs setting. This leads to fully implementable numerical scheme.

Joint work with Goncalo dos Reis (Univeristy of Edinburgh), Arnaud Lionnet (University of Oxford) and Plamen Turkedjiev (Ecole Polytechnique).

C6 - December 20, 14:30 – 15:00

On a SDE with no polynomial convergence rate for strong approximation at the final time

Larisa Yaroslavtseva
University of Passau, Germany
larisa.yaroslavtseva@uni-passau.de

We consider the problem of strong approximation of the solution of a stochastic differential equation (SDE) at the final time based on point evaluations of the driving Brownian motion at a uniform grid. We present an example of a SDE with smooth and bounded coefficients for which no sequence of such approximations can achieve a polynomial rate of convergence. This generalizes a result from [1], which only covers the Euler scheme.

[1] Hairer,M., Hutzenthaler,M., Jentzen,A., Loss of regularity for Kolmogorov equations, Annals of Probability (to appear).

Joint work with Thomas Mueller-Gronbach (University of Passau, Germany).

C6 - December 20, 15:00 – 15:30

Explicit numerical schemes for SDEs driven by Levy noise and for Stochastic Evolution Equations

Sotirios Sabanis
University of Edinburgh, UK
s.sabanis@ed.ac.uk

The idea of 'tamed' Euler schemes, which was pioneered by Hutzenthaler, Jentzen and Kloeden [1] and Sabanis [2], led to the development of a new generation of explicit numerical schemes

- for SDEs driven by Levy noise with superlinear coefficients and,

- for stochastic evolutions equations with super-linearly growing operators appearing in the drift.

Moreover, high order schemes (such as Milstein) are established (with optimal rates of convergence) by the natural extension of the aforementioned ideas. Theoretical results on this topic along with relevant simulation outputs will be presented during this talk.

[1] M. Hutzenthaler, A. Jentzen, P.E. Kloeden, Strong convergence of an explicit numerical method for SDEs with non-globally Lipschitz continuous coefficients. Ann. Appl. Probab. 22 (2012) 1611–1641.

[2] S. Sabanis, A note on tamed Euler approximations, Electron. Commun. Probab. 18 (2013), no. 47, 1—10.

Joint work with Istvan Gyongy (University of Edinburgh), David Siska (University of Edinburgh), Chaman Kumar (University of Edinburgh) and Konstantinos Dareiotis (University of Edinburgh).

C6 - December 20, 15:30 – 16:00

A perturbation formula as universal tool for strong approximations of stochastic differential equations

Martin Hutzenthaler
University of Duisburg-Essen, Germany
martin.hutzenthaler@uni-due.de

The main object of this talk is a pathwise perturbation formula for stochastic differential equations (SDEs) which expresses the distance between the solution and any Itô process in terms of the distances of the local characteristics. Together with suitable integrability properties, this is a convenient tool for proving strong convergence rates of various approximations of SDEs. For example, this yields a sufficient condition for local Lipschitz continuity in the strong sense in the initial value, a sufficient condition for explicit numerical approximations of finite-dimensional SDEs and a sufficient condition for spatial discretizations of nonlinear SPDEs. We illustrate these conditions with example SDEs from finance, physics and biology.

Joint work with Sonja Cox (University of Amsterdam, Netherlands), Arnulf Jentzen (ETH Zurich, Switzerland) and Xiaojie Wang (Changsha, China).

C6 - December 20, 16:00 – 16:30

How to simulate stochastic differential equations without discretizing time

Nawaf Bou-Rabee
Rutgers, USA
nawaf.bourabee@rutgers.edu

Numerical methods for SDEs are primarily based on pathwise approximation, basically if the Brownian motion is given in advance then the solution to the SDE (in a pathwise sense) looks like the solution to an ODE. Since the ODE solution is unique and smooth, it makes sense to discretize in time. In fact, solving the ODE is quite often thought of as an interpolation problem, where the largest time step is used to fit the approximation to the solution. The situation in the SDE context is a bit different: the solution to the SDE is continuous, but not differentiable in general, and an ensemble of trajectories originates from any initial point. Passing to the continuous limit is also nontrivial in the SDE context because existence of a pathwise unique solution imposes stringent conditions on the coefficients and domain of the SDE, which can be difficult to satisfy, and in fact, are not needed in practice. In actual computations one mainly considers the approximation of statistics of the law of the solution (mean first passage times, multi-time expectations, invariant measures, etc.), which is the main aim of SDE solvers, and the existence of a pathwise unique solution is not necessary for convergence in law. In this talk I will numerically illustrate why standard integrators are sometimes inconvenient to use, and present a different approach motivated by the issues encountered when simulating SDEs coming from quantitative finance, population dynamics, chemical kinetics, epidemiology, and polymeric fluids. This talk is based on joint work with Eric Vanden-Eijnden at NYU.

Joint work with Eric Vanden-Eijnden (NYU).

C6 - December 20, 17:00 – 17:30

Weak approximation of the Heston model: non-smooth payoffs

Andreas Neuenkirch
University of Mannheim, Germany
neuenkirch@kiwi.math.uni-mannheim.de

In this talk, we will study the weak approximation of the Heston price process for payoff functions, which are only measurable and bounded. The main tool for our analysis will be the explicit knowledge of the characteristic function of the Heston price, since we can not rely on the seminal work of Bally and Talay (1995). The latter requires smooth coefficients and Gaussian tails for the underlying SDE, which is not fulfilled for the Heston model.

Joint work with Martin Altmayer (University of Mannheim, Germany).

C6 - December 20, 17:30 – 18:00

On a mild Ito formula for stochastic partial differential equations (SPDEs) and on weak convergence rates for SPDEs with nonlinear diffusion coefficients

Arnulf Jentzen
ETH Zurich, Switzerland
arnulf.jentzen@sam.math.ethz.ch

Strong convergence rates for (temporal, spatial, and noise) numerical approximations of semilinear stochastic evolution equations (SEEs) with smooth and regular nonlinearities are well understood in the scientific literature. Weak convergence rates for numerical approximations of such SEEs have been investigated since about 11 years and are far away from being well understood: roughly speaking, no essentially sharp weak convergence rates are known for parabolic SEEs with nonlinear diffusion coefficient functions; see Remark 2.3 in [A. Debussche, Weak approximation of stochastic partial differential equations: the nonlinear case, Math. Comp. 80 (2011), no. 273, 89–117] for details. In this talk we solve the weak convergence problem emerged from Debussche's article and establish esssentially sharp weak convergence rates for different type of temporal and spatial numerical approximations of semilinear SEEs with nonlinear diffusion coefficient functions. Our solution to the weak convergence problem does not use Malliavin calculus. Rather, a key ingredient in our solution to the weak convergence problem emerged from Debussche's article is a new – somehow mild – Ito type formula for solutions and numerical approximations of semilinear SEEs.

Joint work with Daniel Conus (Lehigh University, USA), Giuseppe Da Prato (Scuola Normale Superiore di Pisa, Italy), Ryan Kurniawan (University of Zurich and ETH Zurich, Switzerland), and Michael Röckner (Bielefeld University, Germany).
