FOCM 2014 - Workshop B7
Symbolic Analysis

B7 - December 15, 14:30 – 14:55

A decision method for integrability of partial differential algebraic Pfaffian systems.

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Let $m, n \in \mathbb{N}$. Let $x_1, \ldots, x_m$ be independent variables and $\mathbf{y} := y_1, \ldots, y_n$ be differential unknowns. For each pair (i,j), $1 \leq i \leq n$, $1 \leq j \leq m$, let f_{ij} be a polynomial in $\mathbb{C}[\mathbf{y}]$. A differential algebraic Pfaffian system is a system of differential equations as follows:

$$\Sigma = \begin{cases} \dfrac{\partial y_i}{\partial x_j} &= f_{ij}(\mathbf{y}), \qquad \text{for } i = 1, \ldots, n \text{ and } j = 1 \ldots m, \\ \mathbf{g}(\mathbf{y}) &= 0 \end{cases}$$

where $\mathbf{g} := g_1, \ldots, g_s$ are polynomials in $\mathbb{C}[\mathbf{y}]$.

In this work we are interested in the integrability of these systems, that is, in the existence of infinitely differentiable functions over an open set $\mathcal{U}$ of $\mathbb{C}^m$ that are solutions of Σ. The classical Frobenius Theorem (1877) establishes conditions for a Pfaffian system, without algebraic constraints, to be completely integrable. We focus on the integrability, not necessarily complete, of systems like Σ.

We associate to each system Σ a strictly decreasing chain of algebraic varieties in $\mathbb{C}^n$ of length at most $n+1$. We prove that a necessary and sufficient condition for the existence of solutions for Σ is that the smallest variety of this chain is nonempty. From this result, we are able to show an effective procedure that allows us to decide whether a Pfaffian system is integrable in triple exponential time in n, the number of unknowns.

Joint work with Gabriela Jeronimo (Universidad de Buenos Aires) and Pablo Solernó (Universidad de Buenos Aires).

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Higher order integrable Lagrangians

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We will show results on the integrability of Euler-Lagrange equations arising from Lagrangians of second order. Integrability will be understood under the symmetry approach of A. Shabat et al. Lagrangian sytems usually yield equations of hyperbolic type, and applying the symmetry approach to hyperbolic equations is quite difficult. But in the case of Euler-Lagrange equations, the symmetry approach is applicable if one uses its generalization to certain non evolutionary equations as reported in R. Hernández Heredero, A. Shabat and V. Sokolov, J. Phys. A: Math. Gen. 36 (2003) L605–L614.

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Symmetry classification of curvature evolutions

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Starting from an action of a Lie-group on a manifold the Fels-Olver moving frame method provides a set of generating invariants together with their syzygies. One of the syzygies gives us the evolution of curvature invariants if the evolution of a curve is specified. Another syzygy gives us an invariant symmetry condition which we utilise to identify integrable curvature evolutions.

Joint work with Evelyne Hubert (INRIA Méditerranée, Sophia Antipolis, France).

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Configuration and differential invariants

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We study the relation between the notions of configuration and differential invariants for a G-manifold. The configuration invariants are G-invariant functions defined in the cartesian powers of the G-manifold. Configurations invariants are simpler to understand and to compute than differential invariants, since the prolongation algorithm is just the repetition of the action and does not involve derivation. There is a simpler version of the Lie-Tresse theorem for configuration invariants. We present a general framework that allows to compute the differential invariants associated to a G-manifold from its configuration invariants.

Joint work with Juan Sebastián Díaz (Universidad Nacional de Colombia - Medellín, Colombia).

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Discrete moving frames with applications

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Lie group based moving frames offer a significant new symbolic technology for the study of differential systems. By taking a sequence of frames, many of the excellent features of frames can be adapted to working with difference problems. Indeed, one can obtain a small set of generators of the algebra of invariants, and recurrence relations playing the role of differential syzygies. As for smooth frames, the relations on the difference invariants can be effectively and efficiently computed, without solving for the discrete frame.

In this talk, I will give an overview of the ideas, and show some applications to the difference calculus of variations.

Joint work with Gloria Mari Beffa (University of Madison Wisconsin, USA), Peter Hydon (University of Surrey, UK) and Linyu Peng (Waseda University, Japan).

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Nonlocal symmetries and formal integrability

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In this talk I introduce a simplified version of the classical Krasil'shchik-Vinogradov geometric theory of nonlocal symmetries and present several applications. For example, the theory can be used to find highly non-trivial explicit solutions and Darboux-like transforms to nonlinear equations such as the Kaup-Kupershmidt equation. I also recall the theory of formal integrability and argue that nonlocal symmetries can be used to uncover formally integrable equations. Finally, I present some classifications of nonlocal symmetries of integrable equations which have been recently found, and propose a generalization of the Krasil'shchik-Vinogradov theory.

This talk is partially based on the following papers:

1. E.G. Reyes, Geometric integrability of the Camassa-Holm equation. Letters in Mathematical Physics 59 (2002), 117–131.

2. E.G. Reyes, Nonlocal symmetries and the Kaup-Kupershmidt equation. Journal of Mathematical Physics 46 (2005), 073507 (19 pages).

3. P. Gorka and E.G. Reyes, The modified Camassa-Holm equation. International Mathematics Research Notices (2011) Vol. 2011, 2617–2649.

4. R. Hernandez-Heredero and E.G. Reyes, Geometric integrability of the Camassa-Holm equation II. International Mathematics Research Notices (2012) Vol. 2012, 3089–3125.

5. E.G. Reyes, Jet bundles, symmetries, Darboux transforms. Contemporary Mathematics 563 (2012), 137–164.

6. P. Gorka and E.G. Reyes, The modified Hunter-Saxton equation. Journal of Geometry and Physics 62 (2012), 1793–1809.

7. P.M. Bies, P. Gorka and E.G. Reyes, The dual modified KdV–Fokas–Qiao equation: geometry and local analysis. Journal of Mathematical Physics 53 (2012), 073710.

8. R. Hernandez-Heredero and E.G. Reyes, Nonlocal symmetries, compacton equations, and integrability. International Journal of Geometric Methods in Modern Physics 10 (2013), 1350046 [24 pages].

9. I.S. Krasil'shchik and A.M. Vinogradov, Nonlocal trends in the geometry of differential equations: Symmetries, conservation laws and Backlund transformations. Acta Appl. Math. 15 (1989), 161–209.

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Fast algorithms for the p-curvature of differential operators

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The p-curvature of a linear differential operator in characteristic p is a matrix that measures to what extent the solution space of the operator has dimension close to its order. We describe a recent algorithm for computing the characteristic polynomial of the p-curvature in time $\tilde{O}(p^{0.5})$. The new algorithm allows to test the nilpotency of the p-curvature for primes p of order 10^6, for which the p-curvature itself is impossible to compute using current algorithms.

Joint work with Xavier Caruso (Université Rennes 1, France) and Éric Schost (University of Western Ontario, Canada).

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Computations with Nested Integrals in Particle Physics

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Nested integrals over rational integrands have already been considered by Kummer and Poincaré. Slightly more general integrands involving roots arise in recent computations in the context of perturbative quantum chromodynamics. We discuss symbolic methods dealing with these nested integrals occurring in generating functions, integral transforms, and convolution integrals.

Joint work with Jakob Ablinger (RISC, Austria), Johannes Blümlein (DESY, Germany) and Carsten Schneider (RISC, Austria).

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Equivalence and Invariants: an Overview

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Two objects are said to be equivalent under a prescribed transformation group if one can be mapped to the other by a group element. In particular, symmetries of an object are just its self-equivalences. The case of submanifolds under Lie group and Lie pseudo-group actions is of particular importance, and Élie Cartan gave a general solution to the equivalence problem that relies on matching the functional interdependencies, or syzygies, among their differential invariants. Cartan's solution has been recast into the method of differential invariant signatures that has broad applicability, including image processing, differential equations, the calculus of variations, control theory, a broad range of mathematical physics, differential geometry, classical invariant theory, and so on.

The goal of this talk is to compare and contrast competing approaches to the equivalence problem and the computation of invariants, concentrating on those involving exterior differential systems, those relying on moving frames, and infinitesimal methods dating back to Lie. Recent developments, practical algorithms and some applications of interest will be mentioned during the lecture.

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q-shift operators in knot theory

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In knot theory, the colored Jones function is a knot invariant which is an infinite sequence of Laurent polynomials. Through its definition by state sums it is known to be q-holonomic, i.e., to satisfy a linear recurrence of the form $c_d f_{n+d} + \cdots + c_0 f_n = 0$, $c_d \neq 0$, whose coefficients $c_0, \ldots, c_d$ are bivariate polynomials in q and q^n. We discuss how symbolic computation supports the investigation of this knot invariant.

Joint work with Stavros Garoufalidis (Georgia Tech).

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Sequences and their valuations

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For $x \in \mathbb{N}$ and a prime p, the p-adic valuation $\nu_p(x)$ is the highest power of p that divides x. This is extended to $\mathbb{Z}$ by ignoring the sign and to $\mathbb{Q}$ by defining $\nu_p(a/b) = \nu_p(a) - \nu_p(b)$.

Given a sequence of rational numbers $\{a_n\}$ and a prime p, the sequence of valuations $\{\nu_p(a_n)\}$ reflects properties in the p-adic field $\mathbb{Q}_p$. Examples include values of polynomials, Stirling numbers, Catalan numbers and other examples coming from Combinatorics.

A variety of experimental examples have shown that often this sequence of valuations can be given the structure of a tree. This talk will discuss these examples and discuss the branch structure. Parts of the problem that can lead to automatization will also be discussed.

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Combinatorics, Number Theory, and Symbolic Analysis

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Partition numbers $p(n)$ give the number of additive decompositions of nonnegative integers. For example, $4 = 3+1 = 2+2 = 2+1+1 = 1+1+1+1$, so $p(4) = 5$. Ramanujan observed that all numbers $p(5n+4)$, $n \geq 0$, are divisible by 5. Recently, in the context of modular forms, Silviu Radu (RISC) has set up an algorithmic machinery to prove such congruences automatically. The talk is about new developments in this area and discusses various connections between combinatorics, number theory, and symbolic analysis.

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Multiple binomial sums

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Multiple binomial sums form a very rich class with a lot of structure, which makes it possible to design specific algorithms that prove or discover closed forms or recurrences. In particular, these sums can be expressed as diagonals of rational functions and recurrences can then be obtained by computing a linear differential equation satisfied by the integral of a rational function. We discuss the complexity aspects of this approach as well as its practical use, and compare it to variants of Zeilberger's algorithm.

Joint work with Alin Bostan (Inria, France) and Pierre Lairez (Inria, France).

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Desingularization of Ore Operators

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Singularities of linear differential operators are points at which the numerical computation of solutions is cumbersome. In some cases, this is unavoidable because there is a solution which has a strange behaviour (e.g. a pole) at this point. But sometimes a singularity of a differential operator is only a "false alarm" and does not really correspond to a singularity of a solution. Such singularities are called apparent. Desingularization algorithms eliminate apparent singularities from a given operator. Such algorithms are known since the 19th century. In the talk, we will present a surprisingly simple desingularization algorithm that works not only for differential operators but for general Ore operators. This is joint work with Shaoshi Chen and Michael Singer (arXiv:1408.5512).

Joint work with Shaoshi Chen (Chinese Academy of Sciences, China) and Michael F. Singer (North Carolina State University, USA).

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Computing the parameterized differential Galois group of a second-order linear differential equation with parameters

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Consider a linear differential equation

$$\frac{\partial^2 Y}{\partial x^2} + r_1 \frac{\partial Y}{\partial x} + r_0 Y = 0,$$

where the coefficients $r_1, r_0 \in \mathbb{C}(t_1, \ldots, t_m, x)$. The parameterized Picard-Vessiot theory developed by Phyllis Cassidy and Michael Singer associates a differential Galois group G to such an equation. In analogy with the classical Picard-Vessiot theory of Kolchin, G measures the differential-algebraic relations amongst the solutions to the equation, with respect to $\frac{\partial}{\partial x}$ as well as $\frac{\partial}{\partial t_1}, \ldots, \frac{\partial}{\partial t_m}$.
Relying on earlier work by Thomas Dreyfus, I will describe a complete set of algorithms to compute G, and how these algorithms lead to a simple procedure to decide whether any of the solutions to the equation are differentially transcendental with respect to one or several of the parametric derivations $\frac{\partial}{\partial t_1}, \ldots, \frac{\partial}{\partial t_m}$.

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Algebraic bivariate hypergeometric Laurent series

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I will present joint work with Eduardo Cattani and Federico Martínez on the algebricity of hypergeometric Laurent series in two variables, associated to Cayley configurations of n lattice configurations in n space. We show that these algebraic series are generated by certain combinatorially defined sums of point residues.

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Invariants of Finite Abelian Groups and their use in Symmetry Reduction of Dynamical Systems

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We describe the computation of rational invariants of the linear action of a finite abelian group in the non-modular case and investigate its use in symmetry reductions of dynamical and polynomial systems. Finite abelian subgroups of $GL(n, K)$ can be diagonalized which allows the group action to be accurately described by an integer matrix of exponents. We can make use of integer linear algebra to compute both a minimal generating set of invariants and the substitution to rewrite any invariant in terms of this generating set. The set of invariants provide a symmetry reduction scheme for dynamical and polynomial systems whose solution set is invariant by a finite abelian group action. A special case of the symmetry reduction algorithm applies to reduce the number of parameters in physical, chemical or biological models.

Joint work with Evelyne Hubert (INRIA Mediterranee, France).

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Section, Invariants and Symmetrization

Evelyne Hubert
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For varieties invariant under the action of a finite group, it is known how to convert a set of generators for the ideal into a set of invariants with the same variety. We offer an analogue construction for algebraic groups of positive dimension.

B7 - Poster

Computing periods of rational integrals

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A period of rational integral is the result of integrating, with respect to one or several variables, a rational function along a closed path. When the period under consideration depends on a parameter, it satisfies a specific linear differential equation called Picard-Fuchs equation. These equations and their computation are important for computer algebra, but also for algebraic geometry (where they contains geometric invariants), in combinatorics (where many generating functions are periods) or in theoretical physics.

I present an efficient algorithm to compute these equations. An implementation, which involves only commutative Groebner bases and linear algebra, is available and has been successfully applied to problems that were previously out of reach.