FOCM 2014 - Workshop C2
Foundation of Numerical PDE's

C2 - December 18, 14:30 – 15:05

Numerical approximation of the time-harmonic Maxwell system using H1-conforming finite elements

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We describe a new approximation technique for the Maxwell eigenvalue problem based on H1-conforming finite elements. While reviewing the relevant properties of the Maxwell operator, we point out the difficulties for H1-conforming finite element methods to produce correct spectral approximations. It turns out that the key idea consists of controlling the divergence of the electric field in a fractional Sobolev space with differentiability index between -1 and -1/2. To illustrate the essence of our method, we first examine a non-implementable scheme with this property. Its implementable version relying on a lagrange multiplier to impose such control on the divergence is then discussed. Finally, we examine the case of heterogeneous media. In this context the method needs to cope, in addition, with electric fields not much more than square integrable.

C2 - December 18, 15:10 – 15:45

Scattering of transient waves by penetrable obstacles

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n this talk we will present some results on models for the scattering of transient linear waves by different types of obstacles: homogeneous isotropic; non-homogeneous; elastic. The coupled systems will be discretized with BEM or BEM-FEM in space, and with some implicit or implicit-explicit time-stepping method. We will discuss general stability properties, how to obtain convergence estimates for the fully discrete problems, as well as some delicate questions on how the stability constants behave as functions of time. The results collect current and previous work with Lehel Banjai, Christian Lubich, Tianyu Qiu, Tonatiuh Sanchez-Vizuet, and Matthew Hassell.

C2 - December 18, 15:50 – 16:25

A posteriori error estimators for weighted norms. Adaptivity for point sources and local errors

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We develop a posteriori error estimates for general second order elliptic problems with point sources in two- and three-dimensional domains. We prove a global upper bound and a local lower bound for the error measured in a weighted Sobolev space. The weight belongs to the Muckenhoupt's class A2. The purpose of the weight is twofold. On the one hand it weakens the norm around the singularity, and on the other hand it strengthens the norm in a region of interest, to obtain localized estimates. The theory hinges on local approximation properties of either Clement or Scott-Zhang interpolation operators, without need of suitable modifications, and makes use of weighted estimates for fractional integrals and maximal functions. Numerical experiments illustrate the excellent performance of an adaptive algorithm with the obtained error estimators.

C2 - December 18, 17:05 – 17:55

Finite element spectral approximation of the curl operator in multiply connected domains

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A couple of numerical methods based on Nedelec finite elements have been recently introduced and analyzed in [1] to solve the eigenvalue problem for the curl operator in simply connected domains. This topological assumption is not just a technicality, since the eigenvalue problem is ill-posed on multiply connected domains, in the sense that its spectrum is the whole complex plane as is shown in [2]. However, additional constraints can be added in order to recover a well posed problem with a discrete spectrum [2,3]. We choose as additional constraint a zero-flux condition of the curl on all the cutting surfaces. We introduce two weak formulations of the corresponding problem, which are convenient variations of those studied in [1]; one of them is mixed and the other a Maxwell-like formulation. We prove that both are well posed and show how to modify the finite element discretization from [1] to take care of these additional constraints. We prove spectral convergence of both discretizations and establish a priori error estimates. We also report numerical tests which allow assessing the performance of the proposed methods.

[1] R. Rodriguez and P. Venegas, Numerical approximation of the spectrum of the curl operator, Math. Comp. (online: S 0025-5718(2013)02745-7).

[2] Z. Yoshida and Y. Giga, Remarks on spectra of operator rot, Math. Z., 204 (1990) 235–245.

[3] R. Hiptmair, P.R. Kotiuga and S. Tordeux, Self-adjoint curl operators, Ann. Mat. Pura Appl., 191 (2012) 431–457.

C2 - December 19, 14:30 – 15:05

Stability of an upwind Petrov-Galerkin discretization of convection diffusion equations

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We study a numerical method for convection diffusion equations, in the regime of small viscosity. We identify a norm for which we have both continuity and an inf-sup condition, which are uniform in mesh-width and viscosity, up to logarithmic terms, as long as the viscosity is smaller that the mesh-width. The analysis allows for the formation of a boundary layer.

C2 - December 19, 15:10 – 15:45

Norms in the analysis of the DPG method with optimal test functions

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Standard analysis of the discontinuous Petrov-Galerkin method (DPG) with optimal test functions is based on a direct relationship between trial and test spaces, and their norms. Depending on the particular problem under consideration, theoretical and practical requirements imply different conditions both for the selection of spaces and for the definition of norms. In this talk, we discuss several cases (like convection-dominated diffusion, non-conforming trace approximation, and hypersingular boundary integral operators) and show how problem-dependent objectives force the selection of norms

Joint work with Leszek Demkowicz (The University of Texas at Austin, USA), Michael Karkulik (Pontificia Universidad Catolica de Chile) and Francisco-Javier Sayas (University of Delaware, USA).

C2 - December 19, 15:50 – 16:25

Numerical analysis of electrorheological fluids

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We study a priori estimates for the finite element solutions of electrorheological fluids. These fluids may change their viscosity significantly if an electrical field is applied. Different from the Newtonian case (like water), the friction depends non-linearly on the symmetric gradient of the velocity (power law ansatz). In the case of electrorheological fluids this power is depending additionally on the applied electrical field. We explain step by step the difficulties of the numerical analysis. We start with the p-Laplace system, continue with the p-Stokes system and finally discuss the system for electrorheological fluids. The last step requires the use of Lebesgue spaces with variable exponents.

C2 - December 19, 17:05 – 17:55

The f-wave propagation algorithm for hyperbolic PDEs

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Finite volume methods for solving hyperbolic PDEs, including nonlinear conservation laws whose solutions contain shock waves, are often based on solving one-dimensional Riemann problems. The wave-propagation algorithms implemented in the Clawpack software package provide a very general and robust approach to defining high-resolution methods that exhibit second-order accuracy in smooth regions of the solution while avoiding nonphysical oscillations near discontinuities. This approach is easily applied also to linear hyperbolic problems that are not in conservation form, and can be extended to two or

three space dimensions by the introduction of "transverse Riemann solvers". These algorithms have also been generalized to the so-called f-wave formulation, in which the flux difference between adjacent cells is decomposed as a linear combination of eigenvectors of suitable flux Jacobian matrices. This approach has advantages in many applications including nonlinear problems with spatially varying flux functions, which arise for example in nonlinear elasticity problems in heterogeneous media. The f-wave approach also allows incorporating source terms directly into the Riemann solver in a natural manner, which is essential for some balance laws where the solution sought is a small perturbation to a nontrivial steady state in which nonzero source terms balance the divergence of the flux. Using the shallow water equations to model the propagation of a tsunami across the ocean is an example where this approach has been critical.

C2 - December 20, 14:30 – 15:05

Quadrilateral Q_k elements and the regular decomposition property

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Let $K \subset \mathbb{R}^2$ be a convex quadrilateral. In [1] the following definition can be found: K satisfies the regular decomposition property with constants $N < \infty$ and $0 < \psi < \pi$ if we can divide K into two triangles along one of its diagonals, called d_1, in such a way that $|d_2|/|d_1| < N$ and the maximum angle of both triangles is bounded by ψ. Moreover, in [1] it is shown that the constant in the estimate of the H_1 norm of the error for the Q_1-Lagrange interpolation can be bounded in terms of N and ψ. In [2] this result is generalized to $W^{1,p}$ for $1 \leq p < 3$, while for $3 \leq p$ it is shown that the constant can be bounded in terms of the minimal and the maximal angle of K. In this talk we show the role of the regular decomposition property in quadrilateral Q_k interpolation for $k \geq 2$.

[1] G. Acosta, R. Duran Error estimates for Q_1-isoparametric elements satisfying a weak angle condition. SIAM J. Numer. Anal. 38, 1073-1088, 2000.

[2] G. Acosta, G. Monzon Interpolation error estimates in $W^{1,p}$ for degenerate Q_1-isoparametric elements. Numer. Math. , 104, pp 129-150, 2006.

Joint work with Gabriel Monzón (Universidad de General Sarmiento, Argentina).

C2 - December 20, 15:10 – 15:45

Multi-dimensional polynomial interpolation on arbitrary nodes

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Polynomial interpolation is well understood on the real line. In multi-dimensional spaces, one often adopts a well established one-dimensional method and fills up the space using certain tensor product rule. Examples like this include full tensor construction and sparse grids construction. This approach typically results in fast growth of the total number of interpolation nodes and certain fixed geometrical structure of the nodal sets. This imposes difficulties for practical applications, where obtaining function values at a large number of nodes is infeasible. Also, one often has function data from nodal locations that are not by "mathematical design" and are "unstructured". In this talk, we present a mathematical framework for conducting polynomial interpolation in multiple dimensions using arbitrary set of unstructured nodes. The resulting method, least orthogonal interpolation, is rigorous and has a straightforward numerical implementation. It can faithfully interpolate any function data on any nodal sets, even on those that are considered singular by the traditional methods. We also present a strategy to choose “optimal” nodes that result in robust interpolation. The strategy is based on optimization of Lebesgue function and has certain highly desirable mathematical properties.

C2 - December 20, 15:50 – 16:25

Asymptotic-preserving and well-balanced uncertainty quantification for kinetic and hyperbolic equations

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In this talk we will study the generalized polynomial chaos (gPC) approach to hyperbolic and kinetic equations with uncertain coefficients/inputs, and multiple time or space scales, and show that they can be made asymptotic-preserving or well-balanced, in the sense that the gPC scheme preserves various asymptotic limits in the discrete space. This allows the implemention of the gPC methods for these problems without numerically resolving (by space, time, and gPC modes) the small scales.
